AP CALCULUS
SUMMER WORK PACKET

DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

AP Calculus AB
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Welcome to AP Calculus AB!

This course is a college-level introductory calculus course. We expect that you will treat this course with the same seriousness and dedication that you would treat a college course.

Your **required** summer work includes the following:

1) **Complete the attached problems**. These will be graded **as an assessment**, so be sure to answer them thoroughly using legible handwriting.

2) Purchase a **new or used graphing calculator** of one of the following models: **TI-84 Plus,** or **TI 84 Plus CE**. Older versions will not have the applications you need. You **must** have a graphing calculator by the first day of class. Non-TI calculators (Casio, HP Prime, etc.), "tech-forward" calculators (TI-89, TI-Inspire, etc.) or graphing calculator apps are not acceptable.


3) Complete the **Calculator Skills** section of the packet. You may not understand what each of the directions means, but we expect that you know how to complete the steps of the calculations by day 1.

4) Purchase an **AP Calculus test prep book**. Kaplan, Princeton Review, 5 Steps to a 5, or Barron's are all good books and are easily available.

5) Start getting used to **completing this work with your cell phone put away**. Once you are in class, if your cell phone is out, you will need to leave the class and will earn a zero for the day.

Sign here to state that you completed these problems independently, without assistance from another person, including or use of the Internet **for answers**. Consider these questions open-note and open-book but not open-friend. Using Khan Academy or similar websites or other AP Calculus prep books to *review content* is acceptable.

_____

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

Name: _____

Algebraic Manipulation: **Simplify each expression**

1) $x\left(3x+2\left(x-(2x+1)\right)\right)$

2) $\frac{(9x^2-3x-2)}{(9x^2-4)} \times \frac{(3x^2-10x-8)}{(27x^3)}$

3) $\frac{x+\frac{1}{y}}{y+\frac{1}{x}}$

Solving Equations: Solve each equation for *x*.

4) $x^4-10x^2+29=8$

5) $x^3-5x^2-4x+20=0$

6) $1 - \frac{4}{x} = \frac{5}{6}$

7) $\frac{x+1}{3x+1} + \frac{2x+1}{3x-2} = -1$

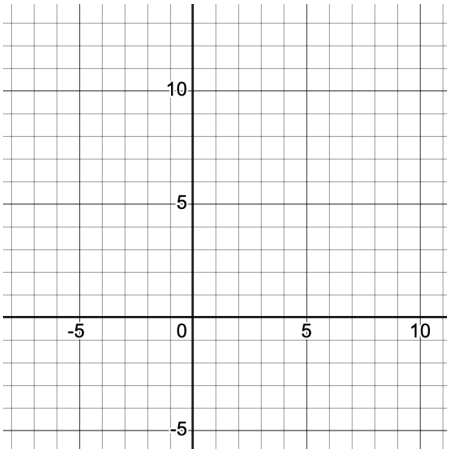
8) $\log_3(x+1) = 3$

9) $2^x \times 2^{x+1} = 16$

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

Sketching graphs:

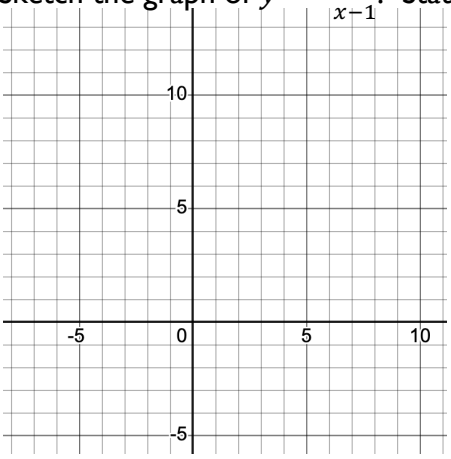
10) Sketch the graph of $y = x^2 + 2x$. State the domain and range.



Domain: _____

Range: _____

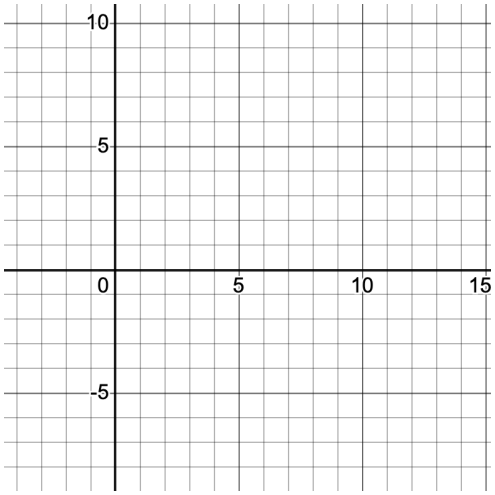
11) Sketch the graph of $y = \frac{2x}{x-1}$. State the domain and range.



Domain: _____

Range: _____

12) Sketch the graph of $y = \ln(x - 2)$. State the domain and range.

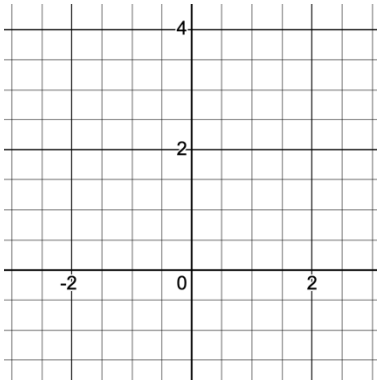


Domain: _____

Range: _____

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

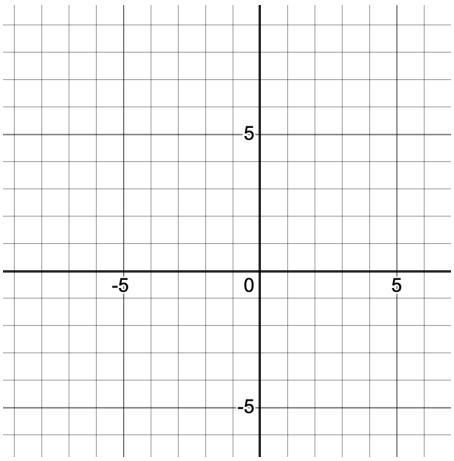
13) Sketch the graph of $2y = \sqrt{36 - 9x^2}$. State the domain and range.



Domain: _____

Range: _____

14) Sketch the piecewise-defined function $h(x) = \begin{cases} \sqrt{3-x}, & x < -1 \\ -3x - 1, & -1 \leq x < 1 \\ x^2, & x \geq 1 \end{cases}$



Finding function Values:

15) If $f(x) = \frac{x-3}{x+4}$, find $f(2)$.

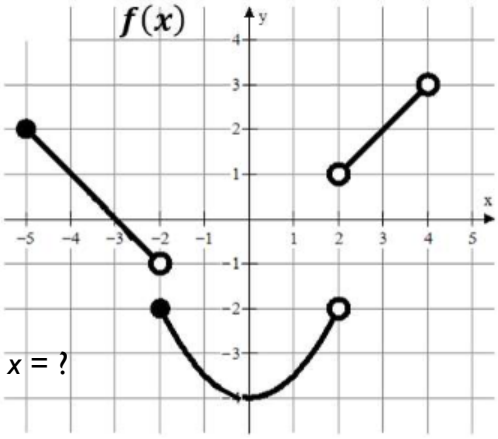
16) If $g(x) = \frac{\cos x}{\cos 2x}$, find $g(\pi)$.

17) If $h(x) = \frac{x+3}{x^2-2x-15}$, find $h(-3)$.

18) If $k(x) = x^2 - 2x + 5$, find $k(x+2)$

19) If $k(x) = x^2 - 2x + 5$, find $k(x+h)$

20) Use the graph of $f(x)$ to find the following.



a. *f*(0)

b. *f*(4)

c. *f*(-1)

d. *f*(-2)

e. *f*(2)

f. *f*(3)

g. *f*(*x*)= 2 when *x* = ?

h. *f*(*x*) = -3 when *x* = ?

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

Equations of Lines:
Write the equation of the line meeting each set of conditions below.

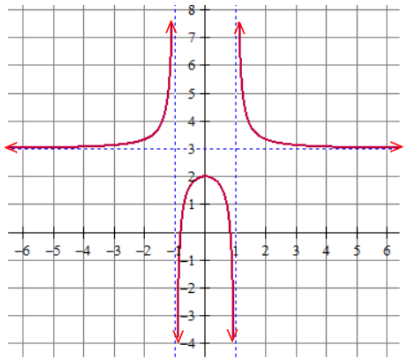
21) slope $m = 3$ and contains the point (4, -2).

22) slope $m = -\frac{3}{2}$ and $f(-5) = 7$.

23) $f(4) = -8$ and $f(-3) = 12$

Graph Analysis:

24) Identify each of the following from the graph below:



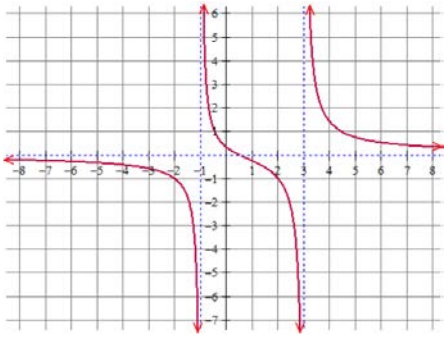
a. Domain:

b. Range:

c. Vertical asymptote(s):

d. Horizontal asymptote(s):

25) Identify each of the following from the graph below:



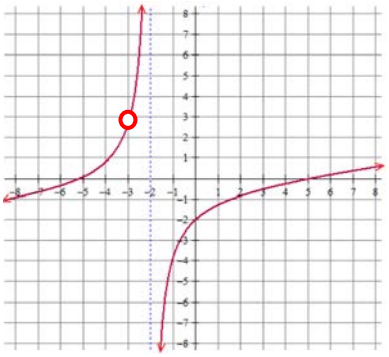
a. Domain:

b. Range:

c. Vertical asymptote(s):

d. Horizontal asymptote(s):

26) Identify each of the following from the graph below:



a. Domain:

b. Range:

c. Vertical asymptote(s):

d. Horizontal asymptote(s):

Exponents:
Rewrite the following using rational exponents.

27) $\sqrt[5]{x^3} + \sqrt[5]{2x}$

28) $\sqrt{x+1}$

29) $\frac{1}{\sqrt{x+1}}$

30) $\frac{1}{\sqrt{x}} - \frac{2}{x}$

31) $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

32) $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Rewrite the following expressions in radical form using positive exponents.

33) $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

34) $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

35) $x^{-\frac{1}{2}}$

36) $(x+4)^{-\frac{1}{2}}$

37) $x^{-2} + x^{\frac{1}{2}}$

38) $2x^{-2} + \frac{3}{2}x^{-1}$

Logarithms:

Solve the following equations:

39) $e^x + 1 = 2$

40) $3e^x + 5 = 8$

41) $e^{2x} = 1$

42) $\ln x = 0$

43) $3 - \ln x = 3$

44) $\ln(3x) = 0$

45) $e^x + xe^x = 0$

46) $e^{2x} - e^x = 0$

47) $x^2 - 3x = 0$

Trigonometry:

Evaluate the following values:

48) $\sin\frac{\pi}{6}$

49) $\cos\frac{\pi}{4}$

50) $\sin 2\pi$

51) $\tan\pi$

52) $\sec\frac{\pi}{2}$

53) $\cos\frac{\pi}{6}$

54) $\sin\frac{\pi}{3}$

55) $\sin\frac{3\pi}{2}$

56) $\tan\frac{\pi}{4}$

57) $\csc\frac{\pi}{2}$

58) $\sin\pi$

59) $\cos\frac{\pi}{3}$

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

Trigonometric Equations

Solve the following trig equations for $0 \leq x \leq 2\pi$.

60) $sin\ x = \frac{1}{2}$

61) $\cos x = -1$

62) $\cos x = \frac{\sqrt{3}}{2}$

63) $2\sin x = -1$

64) $\cos x = \frac{\sqrt{2}}{2}$

65) $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

66) $\tan x = 0$

67) $\sin(2x) = 1$

68) $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

AP Calculus AB
Summer Work Packet – DUE THE LAST DAY OF THE FIRST WEEK OF SCHOOL

Composition of Functions and Inverse Functions

If: $f(x) = \{(3,5),(2,4),(1,7)\}$
$h(x) = \{(3,2),(4,3),(1,6)\}$
$g(x) = \sqrt{x-3}$
$k(x) = x^2 + 5$

Determine each of the following:

69) $(f+h)(1)$

70) $(k-g)(5)$

71) $f(h(3))$

72) $g(k(7))$

73) $g(g(9))$

74) $f^{-1}(4)$

75) $k^{-1}(x)$

Calculator Skills:

Watch this video on calculator functions for the AP Calculus AB exam (Google "Algebros Calculator Skillz"). Bookmark this video so that you can refer to it throughout the year. There will likely be some functions and calculations that you have never heard. That is fine. Right now, we are working on becoming comfortable with your calculator.

After you have watched the video, complete the notes and problems in the "Calculator Skillz" section of the summer packet. This section will count for a separate grade than will the math work in problems #1 – 75.

Calculator Skillz

NOTES

CALCULUS

Write your questions here!

Calculator is always in RADIANS!

CALCULATOR TIPS

1. Use the calculator when it is faster
2. Justify your calculator work on Free Response
3. Do NOT round until the very end
4. Round/Truncate to 3 decimal places

$x = 43.58277289$ $x = -0.49927438$

Common Mistakes

Evaluate

$f(x) = x^2 - 4$ at $x = -3$

$f(x) = \frac{x}{2\pi}$ at $x = 7$

$f(x) = x^9 - 4$ at $x = 47$

Trig Functions

Evaluate

$f\left(\frac{\pi}{5}\right) = \csc\theta$

$f\left(\frac{2}{3\pi}\right) = \sin^2\theta$

Window, Trace, Table, ZStand and ZTrig

Evaluate

$f(x) = x^2 - 4$ at $x = 3.2$

$f(\theta) = \tan^{-1}(\theta)$ at $\theta = \pi$

ZFit, Finding Extrema and Roots

Find all Max/Min

$f(x) = x^4 - 3x^3 + x + 3$

Find the zeros

$f(x) = x^4 - 3x^3 + x + 3$

Finding Point of Intersection

Solve

$y = x^3 + 3x - 4$
$y = -x^2 + 4$

Solve Equations

Solve

$x^3 + 3x - 4 = 5$

STORE and RECALL

If $x = \sin\left(\frac{\pi}{7}\right)$, find $3^x - 2\sqrt{x} - 4x$

$f(x) = x^4 + 3x - 4$
$f(x) = -x^2 + 4$

The *x* coordinate of the left point of intersection is A.
The *x* coordinate of the right point of intersection is B.

Find $A + B$

Window for Word Problems

Methane is produced in a cave at the rate of $r(t) = e^{\sin\left(\frac{\pi}{4}t\right)}$ liters per hour at time *t* hours. The initial amount of methane in the cave at time $t = 0$ is 20 liters. At $t = 8$ hours, a pump begins to remove the methane at a constant rate of 1.5 liters per hour.

At what time *t* during the time interval $0 \le t \le 8$ hours is the amount of methane increasing most rapidly?

SUMMARY:

Now, summarize your notes here!

You are allowed to use a graphing calculator for 1-21

Find all extrema and roots for each function.

1. $y = -\frac{9}{10}x^3 - \frac{3}{4}x^2 + 2x + 1$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

2. $f(x) = \frac{e^x - 1}{x^2 - 4}$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

Solve the systems of equations by graphing.

5. $y = -\ln(2x - 1) + 3$
   $y = e^{\frac{2}{3}x} - 2$

6. $y = \sqrt{x^2 - 4}$
   $y = \tan^{-1}(x) + 3$

Evaluate the function at the given point.

9. $f(x) = e^{x^2 - 1}$ at $x = e$

10. $y = \sec(x) + 5x$ at $x = \frac{\pi}{5}$

11. $f(x) = 3x\sqrt{x^2 + 5}$ at $x = \pi$

12. $y = 2\sin^2(x) + \tan(2x)$ at $x = \frac{\pi}{3}$

Use the STORE feature to evaluate the following.

13. STORE $x = \cot\left(\frac{\pi}{9}\right)$ and use RECALL to find

$$\sqrt{x} + \ln(2x) - e^x$$

14. STORE $x = e^{\pi}$ and use RECALL to find

$$4x - 2\sqrt{x^2 + 1} + 2^x$$

15. Solve the system of equations below. STORE the *x* coordinate of the left point of intersection as A. STORE the *x* coordinate of the right point of intersection as B.

$y = \sin^2(x^2) + 1$
$y = -|2x + 1| + 2.5$

Use RECALL to find $A - B$

16. STORE the *x* coordinate of the maximum point as A. STORE the *x* coordinate of the minimum point as B.

$y = -\frac{2}{5}x^3 - 2x^2 + x + 7$

Use RECALL to find $A - B$

State the WINDOW that allows you to view the function. Answer the question.

17. A tortoise runs along a straight track, starting at position $x = 0$ at time $t = 0$. The tortoise has a velocity of $v(t) = \ln(1 + t^2)$ inches per minute, where *t* is measured in minutes such that $0 \le t \le 15$.

What is the tortoise's velocity at $t = 2.5$?

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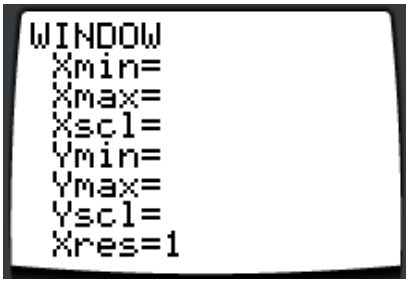
18. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 30-day period. The rate at which the height of the water is rising in the can is given by $s(t) = 2\sin(0.03t) + 1.5$ where $s(t)$ is measured in millimeters per day and *t* is measured in days.



When will the rate of change of the height be 2 mm/day?

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19. For $0 \le t \le 6$, a particle is moving along the *x*-axis. The particle's position, $x(t)$, is not explicitly given. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ in units per second^2.

(NOTE: Acceleration can be positive or negative!)

What is the particle's maximum acceleration?

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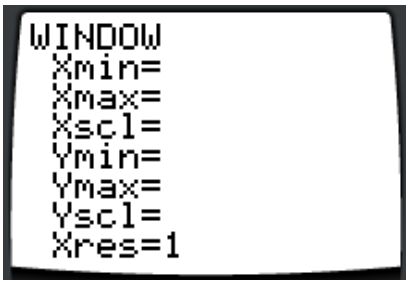
20. The temperature on New Year's Day in Mathlandia was given by by $T(H) = -5 - 10\cos\left(\frac{\pi H}{12}\right)$ where *T* is the temperature in degrees Fahrenheit and *H* is the number of hours from midnight $0 \le H \le 24$.



Find $T(12)$ and explain what it means in this context.

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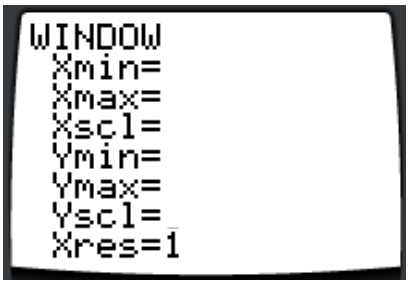
21. A hospital patient is receiving a drug on an IV drip. The rate at which the drug enters the body is given by $E(t) = \frac{4}{1+e^{-t}}$ cubic centimeters per hour. The rate at which the body absorbs the drug is given by $D(t) = 3^{\sqrt{t}-1}$ cubic centimeters per hour. The IV drip starts at time $t = 0$ and continues for 8 hours until time $t = 8$.



Is the amount of drug in the body increasing or decreasing at $t = 6$?

You are allowed to use a graphing calculator for 1-4

MULTIPLE CHOICE

1. Find the value of x for which the graphs of $f(x) = \frac{1}{2}e^{x-4}$ and $g(x) = 3\sqrt[3]{x}$ have $f(x) = g(6)$.

(A) -1.761
(B) 0.35
(C) 2.134
(D) 5.451
(E) 6.389

2. Find the minimum value of the function $f(x) = \ln(x) + \sin(x)$ on the interval $\left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$.

(A) 0.465
(B) 0.526
(C) 0.785
(D) 1.145
(E) 1.605

3. If $f(x) = -\frac{x^2}{x^3-8}$, how many values of c such satisfy the condition $f(c) = 0$?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

4. Which of the following statements about the function $y = x^3(3 - x)$ is true?

I. The function has an absolute maximum.
II. The function has an absolute minimum.
III. The function has a relative minimum.

(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

 **You are allowed to use a graphing calculator** 

FREE RESPONSE

Your score: ____ out of 5

An online retailer has a warehouse that receives packages that are later shipped out to customers. The warehouse is open 12 hours per day. On one particular day, packages are delivered to the warehouse at a rate of $D(t) = 300\sqrt{t} - 3t^2 + 75$ packages per hour. Packages are shipped out at a rate of $S(t) = 60t + 300\sin\left(\frac{\pi}{6}t\right) + 300$ packages per hour. For both functions, $0 \leq t \leq 12$, where *t* is measured in hours. At the beginning of the workday, the warehouse already has 4000 packages waiting to be shipped out.

1. What is the rate of change of the number of packages in the warehouse at time $t = 10$?

2. What is the rate of change of packages shipped out of the warehouse when the rate of change of packages delivered to the warehouse on this day is a maximum?

3. During what time interval(s) is the rate of packages being delivered to the warehouse greater than rate of packages being shipped out of the warehouse?