Washington Latin Public Charter School Summer Math Problem Set for Rising Calculus Students Required of Students Enrolled in Calculus for the 2022-23 Academic Year

Name:

## **DIRECTIONS (please read the directions carefully):**

*Important Note:* Questions containing **HPC** are intended for students who have previously taken Honors Pre-Calculus. Students coming from regular Pre-Calculus are welcome to attempt HPC questions but are not required to answer these questions.

- Please be prepared to have this problem set completed and ready to be handed in on the first day of classes in August.
- This problem set is required of all students who will be enrolled in Calculus in the fall of 2022.
- These practice problems serve as important preparation for a successful experience in Calculus.
- Your ability to work through these problems successfully gives us some insight into how prepared you are for a successful experience in Calculus.
- Please do all of the problems on your own and, if necessary, receive only limited assistance from other people.
- If someone else does most of the work for you, then we will not get an accurate assessment of your knowledge and abilities.
- You can complete this problem set comfortably if you work about 10 problems per day, in which case you would finish the packet in about 10 days.
- This problem set contains a total of 103 questions.
- This problem set contains three types of questions:
  - 1) Student Selected Response Section I (True or False)
  - 2) Student Selected Response Section II (Multiple Choice)
  - 3) Student Generated Free Response Section III
- You are not required to show work for the True/False and Multiple Choice questions.
- You must show your work (thought process) on the Student-Generated Free-Response Questions in order to receive credit.
- The work (steps) that you show is (are) as important as the final answer you give.
- Please write all relevant work clearly within the area provided for each question.
- Please DO NOT write work on other sheets of paper other than these pages.

## **Student Selected Response Section I (True or False)**

- 1. **True or False:** The expression  $(2x-3)^2$  simplifies to  $4x^2+9$
- 2. True or False: The expression  $(x-3y)^3$  simplifies to  $x^3-9x^2y+27xy^2-27y^3$ .
- 3. True or False: The expression  $-6^2$  is equivalent to 36.
- 4. **True or False:** If  $P(t) = \frac{100}{1+3e^{-0.4t}}$ , then P(0) = 25.
- 5. True or False:  $\sqrt{6} \cdot \sqrt{30} = 6\sqrt{5}$ .
- 6. **True or False:** The line containing the points A(-5,7) and A(-5,-2) has equation x = -5.
- 7. True or False:  $\frac{3}{-2x+1} = \frac{-3}{2x-1}$ .
- 8. True or False: If  $\frac{k}{t} = 3m$ , then  $\frac{k}{m} = 3t$
- 9. True or False: Given that x > 0 and y > 0,  $\sqrt{x^2 + y^2} = x + y$

- 10. True or False: Given that x > 0,  $\sqrt{18x^3} = 3x\sqrt{2x}$
- 11. **True or False:** Solving the equation  $x^2 + 100 = 0$  results in the solution set  $\{10i, -10i\}$
- 12. **True or False:**  $\sqrt{x^2 10x + 25} = |x 5|$
- 13. True or False:  $27^{\frac{2}{3}} = \sqrt[3]{27^2} = (\sqrt[3]{27})^2 = (3)^2 = 9$

14. True or False: 
$$\frac{6\sqrt{10}}{2} = 3\sqrt{5}$$

- 15. True or False: Subtracting 2x-5 fro 8x+7 results in 6x-2
- 16. True or False: The two acute angles in a right triangle are always complimentary.
- 17. True or False: If the hypotenuse in a right triangle has length  $\sqrt{20}$  inches and one leg has length  $\sqrt{5}$  inches, then the other leg has length  $\sqrt{15}$  inches.

18. **True or False:**  $6-2(5+3)+40\div10\cdot2=34$ 

19. True or False: The solution set to the equation  $\frac{5x-70}{x-14} = 5$  is all real numbers.

20. True or False: The solution set to the equation  $\frac{6x-78}{x-13} = 7$  is x = 13.

21. True or False: The solution set to the equation  $\frac{6}{x+2} = \frac{9}{x+3}$  is x = 0.

22. True or False: The expression  $x^{2y} \cdot x^{3y}$  is equivalent to  $x^{5y}$ 

23. True or False: The expression  $(x^{2y})^{3y}$  is equivalent to  $x^{6y^2}$ 

24. True or False:  $\log_{10} 5 + \log_{10} 20 = 2$  HPC

25. True or False:  $\log_9 27 = \frac{3}{2}$  HPC

- 26. True or False: The solution set to  $\log x \ge 0$  is  $x \ge 1$  HPC
- 27. True or False: The equation  $\log_{b} a = x$  is equivalent to the equation  $a = b^{x}$  HPC
- 28. True or False:  $\ln(e^{2x}) = 2x$  HPC
- 29. True or False:  $e^{\ln 7} = 7$  HPC

30. True or False: 
$$\tan\theta = \frac{\cos\theta}{\sin\theta}$$
 HPC

## **Student Selected Response Section II (Multiple Choice)**

- 32. Solving 3-2x > 11 results in the solution set a. x < -4 b. x > -4 c. x < 4 d. x > -7 e. x < -7

33. The expression 
$$3\sqrt{48} + 2\sqrt{75}$$
 is equivalent to  
a.  $5\sqrt{123}$  b.  $14\sqrt{6}$  c.  $14\sqrt{3}$  d.  $22\sqrt{6}$  e.  $22\sqrt{3}$ 

34. The expression 
$$3^{\frac{1}{2}} \cdot 3^{\frac{1}{3}}$$
 simplifies to  
a.  $9^{\frac{5}{6}}$  b.  $3^{\frac{2}{6}}$  c.  $9^{\frac{2}{5}}$  d.  $9^{\frac{1}{6}}$  e.  $3^{\frac{5}{6}}$ 

35. The rational expression 
$$\frac{x^2+16}{x-4}$$
 is equivalent to  
a.  $x-4$  b.  $x+4$  c.  $4x$  d. none of these

36. The rational expression 
$$\frac{x^2+5x+14}{x+2}$$
 is equivalent to  
a.  $x+2$  b.  $x+12$  c.  $6x+7$  d.  $x+7$  e. none of these

37. The domain of the function 
$$f(x) = \ln x$$
 is HPC  
a.  $x \ge 1$  b.  $x > 1$  c.  $x > 0$  d.  $x \ge 0$  e.  $(-\infty, \infty)$ 

38. The range of the quadratic function 
$$f(x) = x^2 - 8x + 12$$
 is  
a.  $x \ge 12$  b.  $x \ge 4$  c.  $x \le -4$  d.  $x \ge -4$  e.  $(-\infty, \infty)$ 

39. If a line has an undefined slope, then the line is<br/>a. verticalb. non-horizontal/non-verticalc. horizontald. undefined

40. Given the function  $f(x) = x^2 + 4x - 5$ , identify the values of x for which f(x) < 0a. -5 < x < 1 b. -1 < x < 5 c. x < -5 or x > 1 d. x < -1 or x > 5



43. The expression  $e^{x+y}$  is equivalent to

a. 
$$e^{xy}$$
 b.  $e^{x} \cdot e^{y}$  c.  $(e^{x})^{y}$  d.  $e^{x} + e^{y}$ 

44. The minimum value of the quadratic function  $f(x)=2x^2+4x-1$  is

a. 
$$y = \frac{1}{2}$$
 b.  $y = 6$  c.  $y = -3$  d.  $y = 3$ 

45. Solving the compound inequality -11 < 3x - 8 < 7 results in the solution set a. -5 < x < 1 b. -1 < x < 5 c. x < -5 or x > 1 d. x < -1 or x > 5 46. An equation of the line containing the points (-1,4) and (3,7) is

a. 
$$y = \frac{3}{2}x + \frac{5}{2}$$
 b.  $y = \frac{3}{4}x + \frac{19}{4}$  c.  $y = \frac{3}{4}x + \frac{37}{4}$  d.  $y = \frac{4}{3}x + 3$ 

47. The solution set to 
$$2\sin\theta - 1 = 0$$
 for  $0 \le \theta < 2\pi$  is **HPC**  
a.  $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$  b.  $\left\{\frac{\pi}{6}, \frac{11\pi}{6}\right\}$  c.  $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$  d.  $\left\{\frac{\pi}{3}, \frac{2\pi}{3}\right\}$ 

48. The solution set for the absolute value equation  $|4_x+5|=2_x+1$  is **HPC** a.  $\{2,-1\}$  b.  $\{2\}$  c.  $\{-1\}$  d.  $\{-2,1\}$ 

49. Factoring the polynomial 
$$6x^2 - 7x - 20$$
 results in  
a.  $(6x-5)(x+4)$  b.  $(2x-5)(3x+4)$  c.  $(2x+5)(3x-4)$  d.  $(6x+5)(x-4)$ 

- 50. The graph of the exponential function  $f(x) = 3(2)^x$  contains the points a. (0,0) & (1,6) b. (0,3) & (1,6) c. (0,2) & (1,5) d. (0,3) & (2,36)
- 51.  $\log_3(m \cdot n^2) = \mathbf{HPC}$ a.  $2\log_3 m + 2\log_3 n$  b.  $\log_3 m - 2\log_3 n$  c.  $2\log_3(m+n)$  d.  $\log_3 m + 2\log_3 n$

52. 
$$\ln x - \ln y =$$
 **HPC**  
a.  $\ln \left(\frac{x}{y}\right)$  b.  $\ln(x - y)$  c.  $\frac{\ln x}{\ln y}$  d.  $\ln(xy)$ 

53. Evaluate 
$$\frac{\ln 64}{\ln 4}$$
 HPC  
a. 16 b. ln16 c. 3 d. 8

54. 
$$\sin^2 \theta + \cos^2 \theta =$$
 **HPC**  
a. 1 b. 0 c.  $\tan^2 \theta$  d. -1

55. The rational function  $f(x) = \frac{10}{x^2 - 4}$  has a horizontal asymptote with equation:

- a. y = 0
- b. x = 2
- c. y = 10
- d. x = -2 & x = 2
- e. There is no horizontal asymptote

56. The rational function  $f(x) = \frac{x^2 - 9}{x^2 - 4x - 21}$  has the following domain (Note:  $\mathbb{R}$  represents the set of all real numbers):

- a.  $D: \mathbb{R}, x \neq -3, x \neq 3$
- b.  $D: \mathbb{R}, x \neq -7, x \neq 3$
- c.  $D: \mathbb{R}, x \neq -3, x \neq 7$
- d.  $D: \mathbb{R}, x \neq -21, x \neq -4$
- e.  $D:\mathbb{R}$

57. The rational function  $f(x) = \frac{10}{x^2 - 4}$  has a vertical asymptote(s) with equation:

- a. y = 0
- b. x = 2
- c. y = 10
- d. x = -2 & x = 2
- e. There is no horizontal asymptote

58. The rational function  $f(x) = \frac{x^2 - 8x + 16}{x^2 - 16}$  has a hole at:

- a. y = -1
- b. y = 1
- c. x = -4
- d. x = 4
- e. There is no hole

59. The rational function  $f(x) = \frac{6x^2 + 3x - 1}{2x^2 - 18}$  has a horizontal asymptote with equation:

- a. y = 6
- b. x = 3
- c. y = 0
- d. y = 3
- e. None of the above
- 60. The solution to  $8 = 5^x$  is x =

a. 
$$\frac{\ln 8}{\ln 5}$$
 b.  $\frac{8}{5}$  c.  $\frac{\ln 5}{\ln 8}$  d.  $\frac{5}{8}$ 

## **Student Generated Free Response Section III**

61. Graph the line with equation 5x - 3y = 15.







63. Write the linear equation  $y = \frac{5}{3}x - 2$  in standard form ax + by = c.

64. Simplify the algebraic expression  $(2xy^3)(4x^2y^3)-11x^3y^6+4x^3y^6$ 

65. Simplify the expression  $\frac{x}{8} - \frac{x-16}{8}$ 

66. Simplify the expression  $\frac{3}{4x} - \frac{5}{x^2}$ 

67. Simplify the expression 
$$\frac{x^2 - 16}{x - 4} \div \frac{x^2 + 12x + 32}{x + 8}$$

68. Simplify the expression (write without negative exponents)  $\frac{17x^4}{85x^{-2}}$ 

69. Solve  $\log_4 x + \log_4 (x-6) = 2$ . **HPC** 

70. Rationalize the denominator in the fraction  $\frac{18}{\sqrt{3}}$ 

71. Rationalize the denominator in the fraction  $\frac{16}{3+\sqrt{5}}$  HPC

72. Circle all of the functions that are odd functions HPC

I. 
$$f(x) = \frac{1}{x}$$
 II.  $f(x) = \cos x$  III.  $f(x) = \sin x$  IV.  $f(x) = x^2 - 1$ 

73. For each relation, state the type of symmetry associated with its graph HPC I. xy=1 II.  $y=x^2$  III.  $x^3+y^3=1$  IV.  $x=y^2$ 



74. For the parabola shown, write a quadratic equation in the form of your choice.

75. Given the two functions f(x) = 2 - x and  $g(x) = x^2 + x$ , evaluate g(x) - f(x)

76. Given the two functions 
$$f(x) = 2 - x$$
 and  $g(x) = x^2 + x$ , evaluate  $f(g(x))$ 

77. Given 
$$f(x) = \frac{3}{2}x + 9$$
, determine an equation for the inverse function,  $f^{-1}(x)$ .

78. Shown to the right is the graph of a function y = g(x)

- a. Evaluate g(0)
- b. Write the range of the function y = g(x)
- c. State the interval on which the function is increasing
- d. State the values of x for which g(x)=0
- e. State the interval on which g(x) > 0
- f. State the maximum value of the function y = g(x)
- 79. Graph the function  $f(x) = \sqrt{x+6}$





80. Graph the function  $f(x) = 2^x$ 



81. Graph the function f(x) = -|x-3|



83. Graph the transformation T(x) = f(-x) of the function shown to the right below.



85. Line A has equation  $y = \frac{-5}{3}x + 4$ . Line B contains the point (5, -4) and is perpendicular to line A. Determine an equation for line B.

86. Solve the equation  $28t^2 + 22t - 30 = 0$ 

87. Solve the equation  $5m^3 - 45m = 0$ 

88. Use the quadratic formula to solve the equation  $x^2 - 6x + 4 = 0$ 

89. Solve  $x^3 - 2x^2 - 12x + 24 = 0$ 

90. Solve the system of equations  $x^2 + y^2 = 25$  and  $y = x^2 - 5$  algebraically. **HPC** 

91. Solve 
$$\sqrt{3x+7} = x+1$$
. **HPC**

92. Solve 
$$\frac{2}{x-3} \ge 1$$
. HPC

93. Given x+1 is a factor of  $g(x) = x^3 - 4x^2 + x + 6$ , find all of the zeros of g(x). HPC

94. Given 
$$g(x) = -3\cos\left[4\left(x - \frac{\pi}{12}\right)\right] + 2$$
, determine each of the following: **HPC**

- a. the amplitude is \_\_\_\_\_
- b. the vertical shift is \_\_\_\_\_
- c. the period is \_\_\_\_\_
- d. the phase shift (horizontal shift) is \_\_\_\_\_
- e. the frequency is \_\_\_\_\_

95. Give an exact value for  $\sin \frac{\pi}{3}$ 

96. Give an exact value for  $\tan \frac{\pi}{2}$ 

97. Give an exact value for 
$$\cos \frac{\pi}{6}$$

98. Give an exact value for  $\sin \frac{\pi}{4}$ 

99. Evaluate  $\sin^{-1}(0)$ 

100. Evaluate  $\cos^{-1}(1)$ 

101. Evaluate  $\tan^{-1}(1)$ 

102. Evaluate  $\sin^{-1}(1)$ 

103. Evaluate  $\cos^{-1}(-1)$