Washington Latin Public Charter School
Summer Math Problem Set for Rising Calculus Students
Required of Students Enrolled in Calculus for the 2021-22 Academic Year

Name:

DIRECTIONS (please read the directions carefully):

Important Note: Questions containing **HPC** are intended for students who have previously taken Honors Pre-Calculus. Students coming from regular Pre-Calculus are welcome to attempt HPC questions but are not required to answer these questions.

- Please be prepared to have this problem set completed and ready to be handed in on the first day of classes in August.
- This problem set is required of all students who will be enrolled in Calculus in the fall of 2019.
- These practice problems serve as important preparation for a successful experience in Calculus.
- Your ability to work through these problems successfully gives us some insight into how prepared you are for a successful experience in Calculus.
- Please do all of the problems on your own and, if necessary, receive only limited assistance from other people.
- If someone else does most of the work for you, then we will not get an accurate assessment of your knowledge and abilities.
- You can complete this problem set comfortably if you work about 10 problems per day, in which case you would finish the packet in about 10 days.
- This problem set contains a total of 103 questions.
- This problem set contains three types of questions:
    1) Student Selected Response Section I (True or False)
    2) Student Selected Response Section II (Multiple Choice)
    3) Student Generated Free Response Section III
- You are not required to show work for the True/False and Multiple Choice questions.
- You must show your work (thought process) on the Student-Generated Free-Response Questions in order to receive credit.
- The work (steps) that you show is (are) as important as the final answer you give.
- Please write all relevant work clearly within the area provided for each question.
- Please DO NOT write work on other sheets of paper other than these pages.

Student Selected Response Section I (True or False)

1. **True or False:** The expression $(2x-3)^2$ simplifies to $4x^2+9$

2. **True or False:** The expression $(x-3y)^3$ simplifies to $x^3-9x^2y+27xy^2-27y^3$.

3. **True or False:** The expression -6^2 is equivalent to 36.

4. **True or False:** If $P(t)=\dfrac{100}{1+3e^{-0.4t}}$, then $P(0)=25$.

5. **True or False:** $\sqrt{6}\cdot\sqrt{30}=6\sqrt{5}$.

6. **True or False:** The line containing the points $A(-5,7)$ and $A(-5,-2)$ has equation $x=-5$.

7. **True or False:** $\dfrac{3}{-2x+1}=\dfrac{-3}{2x-1}$.

8. **True or False:** If $\dfrac{k}{t}=3m$, then $\dfrac{k}{m}=3t$

9. **True or False:** Given that $x>0$ and $y>0$, $\sqrt{x^2+y^2}=x+y$

10. **True or False:** Given that $x>0$, $\sqrt{18x^3}=3x\sqrt{2x}$

11. **True or False:** Solving the equation $x^2+100=0$ results in the solution set $\{10i,-10i\}$

12. **True or False:** $\sqrt{x^2-10x+25}=|x-5|$

13. **True or False:** $27^{\frac{2}{3}}=\sqrt[3]{27^2}=\left(\sqrt[3]{27}\right)^2=(3)^2=9$

14. **True or False:** $\dfrac{6\sqrt{10}}{2}=3\sqrt{5}$

15. **True or False:** Subtracting $2x-5$ fro $8x+7$ results in $6x-2$

16. **True or False:** The two acute angles in a right triangle are always complimentary.

17. **True or False:** If the hypotenuse in a right triangle has length $\sqrt{20}$ inches and one leg has length $\sqrt{5}$ inches, then the other leg has length $\sqrt{15}$ inches.

18. **True or False:** $6-2(5+3)+40\div10\cdot2=34$

19. **True or False:** The solution set to the equation $\dfrac{5x-70}{x-14}=5$ is all real numbers.

20. **True or False:** The solution set to the equation $\dfrac{6x-78}{x-13}=7$ is $x=13$.

21. **True or False:** The solution set to the equation $\dfrac{6}{x+2}=\dfrac{9}{x+3}$ is $x=0$.

22. **True or False:** The expression $x^{2y}\cdot x^{3y}$ is equivalent to x^{5y}

23. True or False: The expression $\left(x^{2y}\right)^{3y}$ is equivalent to x^{6y^2}

24. **True or False:** $\log_{10}5+\log_{10}20=2$ **HPC**

25. **True or False:** $\log_{9}27=\dfrac{3}{2}$ **HPC**

26. **True or False:** The solution set to $\log x \geq 0$ is $x \geq 1$ **HPC**

27. **True or False:** The equation $\log_b a = x$ is equivalent to the equation $a = b^x$ **HPC**

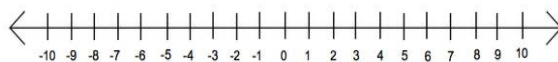28. **True or False:** $\ln\left(e^{2x}\right) = 2x$ **HPC**

29. **True or False:** $e^{\ln 7} = 7$ **HPC**

30. **True or False:** $\tan\theta = \dfrac{\cos\theta}{\sin\theta}$ **HPC**

Student Selected Response Section II (Multiple Choice)

31. The set of numbers graphed on the real number line is represented by the set:

    a. $-1 \geq x < 6$ b. $-1 < x \leq 6$ c. $-1 \leq x \leq 6$ d. $-1 < x > 6$ e. $-1 < x < 6$

32. Solving $3 - 2x > 11$ results in the solution set

   a. $x < -4$ b. $x > -4$ c. $x < 4$ d. $x > -7$ e. $x < -7$

33. The expression $3\sqrt{48}+2\sqrt{75}$ is equivalent to

    a. $5\sqrt{123}$ b. $14\sqrt{6}$ c. $14\sqrt{3}$ d. $22\sqrt{6}$ e. $22\sqrt{3}$

34. The expression $3^{\frac{1}{2}}\cdot 3^{\frac{1}{3}}$ simplifies to

    a. $9^{\frac{5}{6}}$ b. $3^{\frac{2}{6}}$ c. $9^{\frac{2}{5}}$ d. $9^{\frac{1}{6}}$ e. $3^{\frac{5}{6}}$

35. The rational expression $\dfrac{x^2+16}{x-4}$ is equivalent to

    a. $x-4$ b. $x+4$ c. $4x$ d. none of these

36. The rational expression $\dfrac{x^2+5x+14}{x+2}$ is equivalent to

    a. $x+2$ b. $x+12$ c. $6x+7$ d. $x+7$ e. none of these

37. The domain of the function $f(x)=\ln x$ is HPC

    a. $x\geq 1$ b. $x>1$ c. $x>0$ d. $x\geq 0$ e. $\left(-\infty,\infty\right)$

38. The range of the quadratic function $f(x)=x^2-8x+12$ is

    a. $x\geq 12$ b. $x\geq 4$ c. $x\leq -4$ d. $x\geq -4$ e. $\left(-\infty,\infty\right)$

39. If a line has an undefined slope, then the line is

    a. vertical   b. non-horizontal/non-vertical   c. horizontal   d. undefined

40. Given the function $f(x)=x^2+4x-5$, identify the values of x for which $f(x)<0$

    a. $-5<x<1$ b. $-1<x<5$ c. $x<-5 \ or \ x>1$ d. $x<-1 \ or \ x>5$

41. Refer to triangle ABC below. $\sin A =$

    a. $\dfrac{4}{5}$ b. $\dfrac{3}{4}$ c. $\dfrac{5}{3}$ d. $\dfrac{5}{4}$ e. $\dfrac{3}{5}$
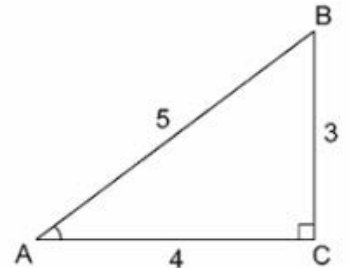
42. Refer to triangle ABC below. $\tan A =$

    a. $\dfrac{5}{4}$ b. $\dfrac{3}{5}$ c. $\dfrac{3}{4}$ d. $\dfrac{5}{3}$ e. $\dfrac{4}{5}$

43. The expression e^{x+y} is equivalent to

    a. e^{xy} b. $e^x \cdot e^y$ c. $\left(e^x\right)^y$ d. $e^x + e^y$

44. The minimum value of the quadratic function $f(x)=2x^2+4x-1$ is

    a. $y=\dfrac{1}{2}$ b. $y=6$ c. $y=-3$ d. $y=3$

45. Solving the compound inequality $-11<3x-8<7$ results in the solution set

    a. $-5<x<1$ b. $-1<x<5$ c. $x<-5 \ or \ x>1$ d. $x<-1 \ or \ x>5$

46. An equation of the line containing the points $(-1,4)$ and $(3,7)$ is

    a. $y = \dfrac{3}{2}x + \dfrac{5}{2}$ b. $y = \dfrac{3}{4}x + \dfrac{19}{4}$ c. $y = \dfrac{3}{4}x + \dfrac{37}{4}$ d. $y = \dfrac{4}{3}x + 3$

47. The solution set to $2\sin\theta - 1 = 0$ for $0 \le \theta < 2\pi$ is **HPC**

    a. $\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}$ b. $\left\{\dfrac{\pi}{6}, \dfrac{11\pi}{6}\right\}$ c. $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}$ d. $\left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right\}$

48. The solution set for the absolute value equation $|4x+5| = 2x+1$ is **HPC**

    a. $\{2,-1\}$ b. $\{2\}$ c. $\{-1\}$ d. $\{-2,1\}$

49. Factoring the polynomial $6x^2 - 7x - 20$ results in

    a. $(6x-5)(x+4)$ b. $(2x-5)(3x+4)$ c. $(2x+5)(3x-4)$ d. $(6x+5)(x-4)$

50. The graph of the exponential function $f(x) = 3(2)^x$ contains the points

    a. $(0,0)$ & $(1,6)$ b. $(0,3)$ & $(1,6)$ c. $(0,2)$ & $(1,5)$ d. $(0,3)$ & $(2,36)$

51. $\log_3(m \cdot n^2) = $ **HPC**

    a. $2\log_3 m + 2\log_3 n$ b. $\log_3 m - 2\log_3 n$ c. $2\log_3(m+n)$ d. $\log_3 m + 2\log_3 n$

52. $\ln x - \ln y =$ **HPC**

    a. $\ln\left(\dfrac{x}{y}\right)$ b. $\ln(x-y)$ c. $\dfrac{\ln x}{\ln y}$ d. $\ln(xy)$

53. Evaluate $\dfrac{\ln 64}{\ln 4}$ **HPC**

    a. 16 b. $\ln 16$ c. 3 d. 8

54. $\sin^2\theta + \cos^2\theta =$ **HPC**

    a. 1 b. 0 c. $\tan^2\theta$ d. -1

55. The rational function $f(x) = \dfrac{10}{x^2 - 4}$ has a horizontal asymptote with equation:

    a. $y = 0$
    b. $x = 2$
    c. $y = 10$
    d. $x = -2$ & $x = 2$
    e.   There is no horizontal asymptote

56. The rational function $f(x) = \dfrac{x^2 - 9}{x^2 - 4x - 21}$ has the following domain (Note: $\mathbb{R}$ represents the set of all real numbers):

    a. $D: \mathbb{R}, x \neq -3, x \neq 3$
    b. $D: \mathbb{R}, x \neq -7, x \neq 3$
    c. $D: \mathbb{R}, x \neq -3, x \neq 7$
    d. $D: \mathbb{R}, x \neq -21, x \neq -4$
    e. $D: \mathbb{R}$

57. The rational function $f(x) = \dfrac{10}{x^2 - 4}$ has a vertical asymptote(s) with equation:

    a. $y = 0$

    b. $x = 2$

    c. $y = 10$

    d. $x = -2 \ \& \ x = 2$

    e.   There is no horizontal asymptote

58. The rational function $f(x) = \dfrac{x^2 - 8x + 16}{x^2 - 16}$ has a hole at:

    a. $y = -1$

    b. $y = 1$

    c. $x = -4$

    d. $x = 4$

    e.   There is no hole

59. The rational function $f(x) = \dfrac{6x^2 + 3x - 1}{2x^2 - 18}$ has a horizontal asymptote with equation:

    a. $y = 6$

    b. $x = 3$

    c. $y = 0$

    d. $y = 3$

    e.   None of the above

60. The solution to $8 = 5^x$ is $x =$

    a. $\dfrac{\ln 8}{\ln 5}$ b. $\dfrac{8}{5}$ c. $\dfrac{\ln 5}{\ln 8}$ d. $\dfrac{5}{8}$

Student Generated Free Response Section III

61. Graph the line with equation $5x - 3y = 15$.
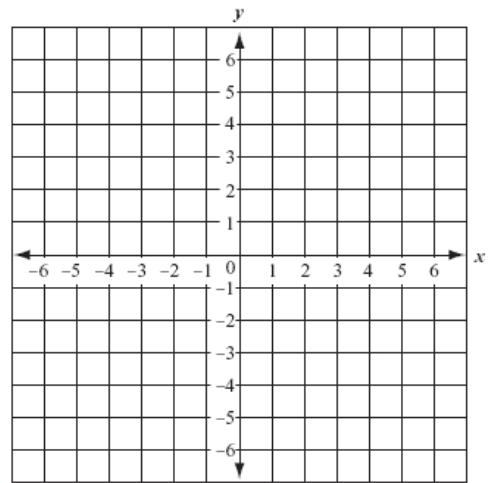
62. Graph the line with equation $y = x$.
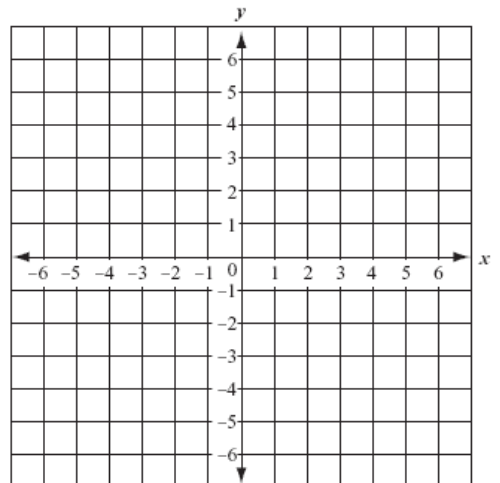
63. Write the linear equation $y = \dfrac{5}{3}x - 2$ in standard form $ax + by = c$.

64. Simplify the algebraic expression $\left(2xy^3\right)\left(4x^2y^3\right) - 11x^3y^6 + 4x^3y^6$

65. Simplify the expression $\dfrac{x}{8} - \dfrac{x - 16}{8}$

66. Simplify the expression $\dfrac{3}{4x} - \dfrac{5}{x^2}$

67. Simplify the expression $\dfrac{x^2-16}{x-4} \div \dfrac{x^2+12x+32}{x+8}$

68. Simplify the expression (write without negative exponents) $\dfrac{17x^4}{85x^{-2}}$

69. Solve $\log_4 x + \log_4(x-6) = 2.$ **HPC**

70. Rationalize the denominator in the fraction $\dfrac{18}{\sqrt{3}}$

71. Rationalize the denominator in the fraction $\dfrac{16}{3+\sqrt{5}}$ **HPC**

72. Circle all of the functions that are odd functions   **HPC**

   I. $f(x) = \dfrac{1}{x}$ II. $f(x) = \cos x$ III. $f(x) = \sin x$ IV. $f(x) = x^2 - 1$

73. For each relation, state the type of symmetry associated with its graph   **HPC**

   I. $xy = 1$ II. $y = x^2$ III. $x^3 + y^3 = 1$ IV. $x = y^2$

74. For the parabola shown, write a quadratic
    equation in the form of your choice.
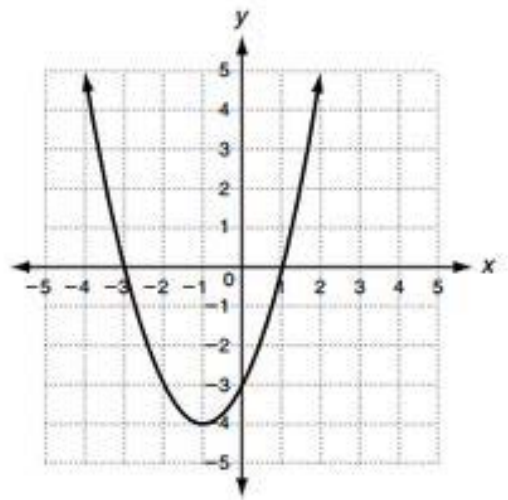
75. Given the two functions $f(x) = 2 - x$ and $g(x) = x^2 + x$, evaluate $g(x) - f(x)$

76. Given the two functions $f(x) = 2 - x$ and $g(x) = x^2 + x$, evaluate $f(g(x))$

77. Given $f(x) = \dfrac{3}{2}x + 9$, determine an equation for the inverse function, $f^{-1}(x)$.

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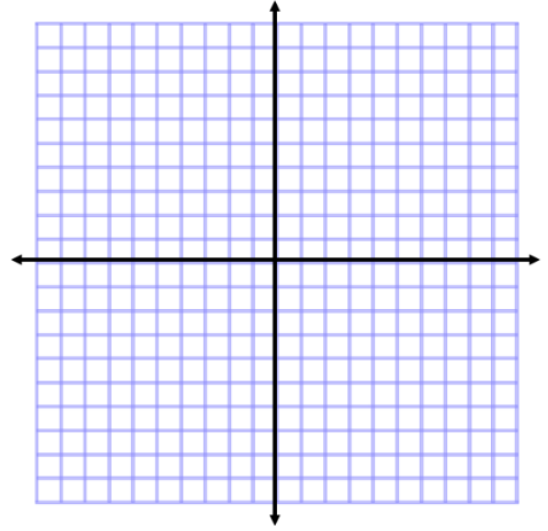78. Shown to the right is the graph of a function $y = g(x)$

   a. Evaluate $g(0)$

   b. Write the range of the function $y = g(x)$

   c. State the interval on which the function is increasing

   d. State the values of x for which $g(x) = 0$

   e. State the interval on which $g(x) > 0$
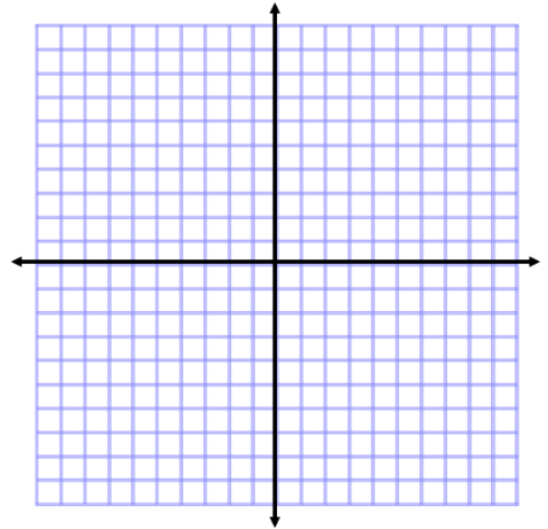
   f. State the maximum value of the function $y = g(x)$
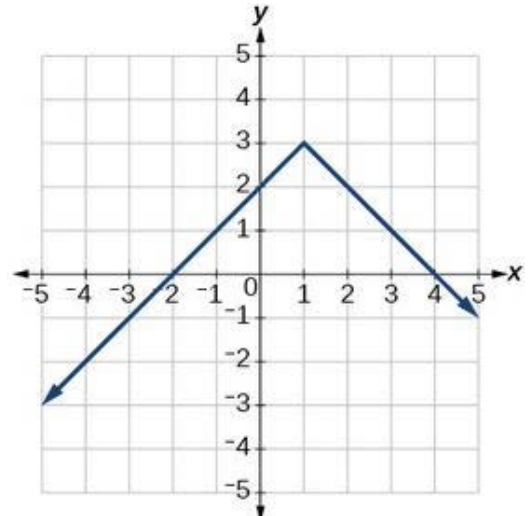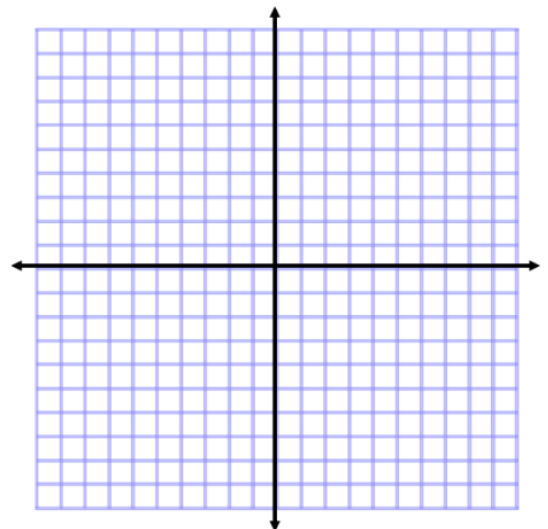
79. Graph the function $f(x) = \sqrt{x+6}$
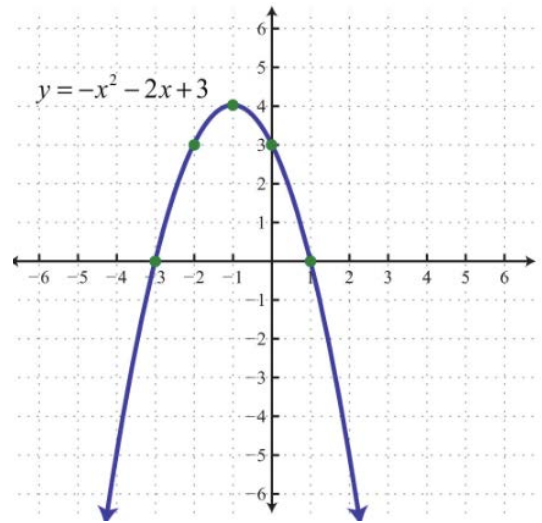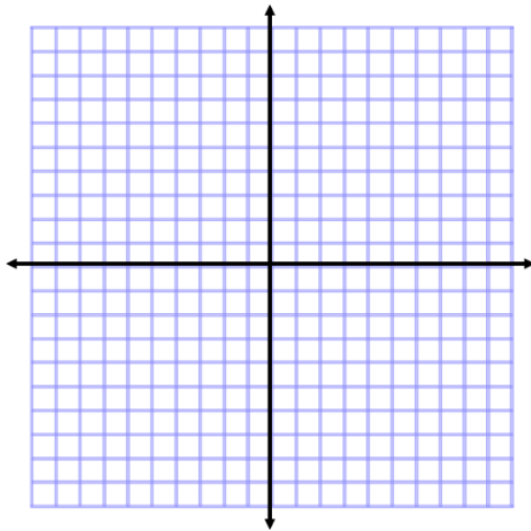
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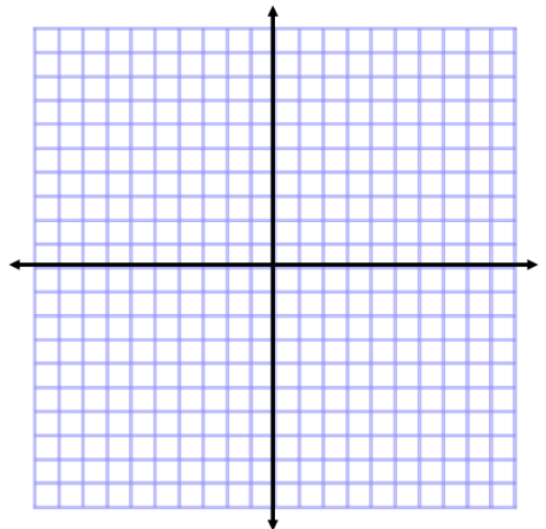
80. Graph the function $f(x) = 2^x$

81. Graph the function $f(x) = -|x - 3|$

82. Graph the function $f(x) = \dfrac{1}{x+4} - 2$

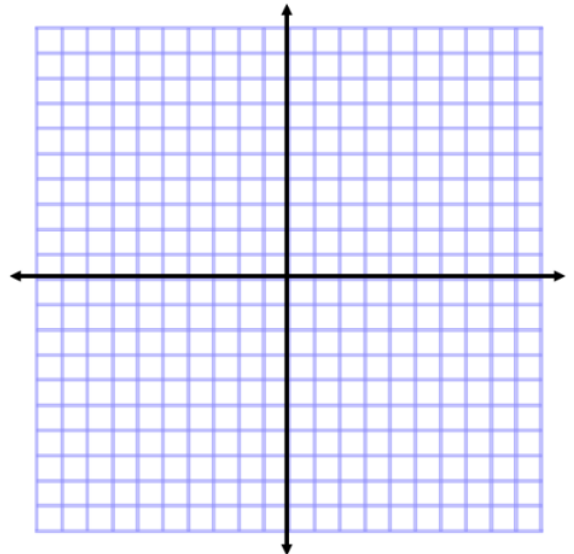83. Graph the transformation $T(x) = f(-x)$ of the function shown to the right below.

$$y = -x^2 - 2x + 3$$

84. Graph the piecewise function

$$f(x) = \begin{cases} x - 3 & \text{if } x > 1 \\ -2 & \text{if } x \leq 1 \end{cases}$$

85. Line A has equation $y = \dfrac{-5}{3}x + 4$. Line B contains the point $(5, -4)$ and is perpendicular to line A. Determine an equation for line B.

86. Solve the equation $28t^2 + 22t - 30 = 0$

87. Solve the equation $5m^3 - 45m = 0$

88. Use the quadratic formula to solve the equation $x^2 - 6x + 4 = 0$

89. Solve $x^3 - 2x^2 - 12x + 24 = 0$

90. Solve the system of equations $x^2 + y^2 = 25$ and $y = x^2 - 5$ algebraically. **HPC**

91. Solve $\sqrt{3x+7} = x+1$. **HPC**

92. Solve $\dfrac{2}{x-3} \geq 1$. **HPC**

93. Given $x+1$ is a factor of $g(x) = x^3 - 4x^2 + x + 6$, find all of the zeros of $g(x)$. **HPC**

94. Given $g(x) = -3\cos\left[4\left(x - \dfrac{\pi}{12}\right)\right] + 2$, determine each of the following: **HPC**

   a. the amplitude is _____

   b. the vertical shift is _____

   c. the period is _____

   d. the phase shift (horizontal shift) is _____

   e. the frequency is _____

95. Give an exact value for $\sin\dfrac{\pi}{3}$

96. Give an exact value for $\tan\dfrac{\pi}{2}$

97. Give an exact value for $\cos\dfrac{\pi}{6}$

98. Give an exact value for $\sin\dfrac{\pi}{4}$

99. Evaluate $\sin^{-1}(0)$

100. Evaluate $\cos^{-1}(1)$

101. Evaluate $\tan^{-1}(1)$

102. Evaluate $\sin^{-1}(1)$

103. Evaluate $\cos^{-1}(-1)$