Geometry

Summer Packet 2021
WLPCS

Welcome to geometry! Geometry is a language of mathematics that describes the world around us. Architects, artists, engineers, physicists, surveyors, and video game designers all use geometry every day. We will learn about where these concepts come from and how to use them. This packet will help you prepare for the upcoming year by drawing on what you already know.

This packet is **entirely assembled** from materials from previous math classes. You should be familiar with each of these problems. The skills in this packet are the ones *you will be using* this upcoming year in geometry, so you need to refresh them before you begin.

If there are any terms, topics, or problems you <u>do not</u> remember, do your best to find instruction online (Khan Academy or Youtube, for example), and try to do as many steps as possible. Do not leave any problems blank. You will receive your first major grade of the year for this packet.

Directions:
- Show your work. The work is not only to show an answer, but to show the reader your sequence of thinking and operations.
- All final expressions or other solutions must be in simplest form.
- Box your answers.

EXAMPLES

EX1 – Simplify the expression (PEMDAS)

$$\frac{(3-13)^2 + 14}{2^3 - 5 \cdot 2} \to \frac{(-10)^2 + 14}{2^3 - 5 \cdot 2} \to \frac{100 + 14}{8 - 5 \cdot 2} \to \frac{100 + 14}{8 - 10} \to \frac{114}{-2} = \boxed{-57}$$

EX2 – Solve the proportion using the "cross multiplication" method

$$\frac{y-3}{2} \times \frac{y}{5} \to 2 \cdot y = 5(y-3) \to 2y = 5y - 15 \to -3y = -15 \to \boxed{y = 5}$$
$$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad -5y \quad -5y \quad\quad\quad\quad \frac{-3y}{-3} = \frac{-15}{-3}$$

EX3 – Find the area

$b = 10.3$ km
$h = 5$ km

rectangle: $A = b \cdot h$

$$A = 10.3 \cdot 5 = \boxed{51.5 \text{ km}^2} \text{ units!}$$

EX4 – Set up an equation and solve
"The sum of five times a number and two is 17"

$$+ \quad 5 \quad \cdot \quad x \quad 2 = 17$$

$$5 \cdot x + 2 = 17$$

$$5x + 2 = 17$$
$$\quad\; -2 \quad\; -2$$
$$\frac{5x}{5} = \frac{15}{5} \to \boxed{x = 3}$$

Properties of Algebra

Review the Properties of Algebra and the provided examples. Then, answer the following questions

Commutative Property $(+, \times)$	**Associative Property** $(+, \times)$	**Identity Property** $(+, \times)$
Order of values does not matter.	Grouping of values does not matter.	Values stay the same after operation.
$$a + b = b + a$$ $$2 + 3 = 3 + 2$$ $$5 = 5$$	$$(a + b) + c = a + (b + c)$$ $$(5 + 1) + 3 = 5 + (1 + 3)$$ $$6 + 3 = 5 + 4$$ $$9 = 9$$	*Additive Identity* (0) $$a + 0 = a$$
$$x \cdot y = y \cdot x$$ $$5 \cdot 4 = 4 \cdot 5$$ $$20 = 20$$	$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$ $$(2 \cdot 4) \cdot 7 = 2 \cdot (4 \cdot 7)$$ $$8 \cdot 7 = 2 \cdot 28$$ $$56 = 56$$	*Multiplicative Identity* (1) $$a \cdot 1 = a$$
Inverse Property $(+, \times)$	**Property of Zero**	**Distributive Property**
Use the opposite (inverse) to eliminate a value (identity).	Multiplying by zero (0) always equals zero (0).	Multiply a value to each value of an expression inside parentheses ().
Additive Inverse *(negetive)* $$a + (-a) = 0$$ $$a - a = 0$$ *Multiplicative Inverse* *(reciprocal)* $$a \cdot \left(\frac{1}{a}\right) = 1$$ $$\frac{a}{a} = 1$$	$$a \cdot 0 = 0$$ $$17 \cdot 0 = 0$$ $$\frac{1}{a} \cdot 0 = 0$$ $$\frac{1}{-4} \cdot 0 = 0$$	$$a(b + c) = ab + ac$$ $$2(x + 7) = 2x + 14$$ $$-5(y - z) = -5y + 5z$$ ** Notice that $-5 \cdot -z = 5z$, because a negative times a negative is positive $$(- \cdot - = +)$$
Reflexive Property	**Symmetric Property**	**Transitive Property**
A value is always equal to itself.	The order of an equation does not matter.	If two objects equal the same value, the objects are equal.
$$a = a$$ $$16 = 16$$ $$5x = 5x$$ $$4y^2 = 4y^2$$	If $a = b$, then $b = a$ If $3 = x$, then $x = 3$ If $5x = 30$, then $30 = 5x$	If $a = b$ and $b = c$, then $a = c$ If $4 + 3 = 7$ and $7 = \sqrt{49}$, then $4 + 3 = \sqrt{49}$ If $x + 5 = y$ and $y = 15$, then $x + 5 = 15$

Match the property from the bank to the corresponding example.

1) _____ - $16 = 16$

2) _____ - $7 + (a + b) = (7 + a) + b$

3) _____ - $(4y + 1) \cdot 0 = 0$

4) _____ - If $5 \cdot 2 = 10$ and $10 = \sqrt{100}$, then $5 \cdot 2 = \sqrt{100}$

5) _____ - $-2(x - y) = -2x + 2y$

6) _____ - $3 \cdot \frac{1}{3} = 1$

7) _____ - If $8x = 24$, then $24 = 8x$

8) _____ - $(2 + 1) + 9 = (1 + 2) + 9$

9) _____ - $6m \cdot 1 = 6m$

A. Commutative Property

B. Associative Property

C. Distributive Property

D. Identity Property

E. Inverse Property

F. Property of Zero

G. Reflexive Property

H. Symmetric Property

I. Transitive Property

Expressions (Order of Operations)

Evaluate each expression using order of operations (PEMDAS).

10) $24 - 4^2 \cdot 3 + 15$

11) $\dfrac{(7 - 2^2) + 17}{-14 + 2 \cdot 5}$

12) $(4 - 7)^2 - 6 \cdot 7 + 20$

13) $2 \left[\dfrac{18 - \left(5 + 3^2 \right)}{7} \right]$

14) $1 + (-2 - 5)^2 + (14 - 17) \cdot 4$

15) $\dfrac{6(2+4) - 1}{2 \cdot 3 + 1}$

16) $\dfrac{17 \cdot 5 - 3 \cdot 5}{3^2 + 1}$

Simplify Expressions (Like Terms)

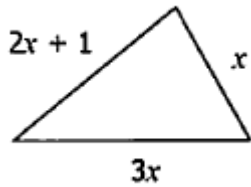Simplify each expression by combining like terms.
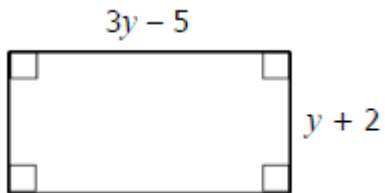
17) $3x + 6x$

18) $7m - 5m - 6$

19) $5x + 7y - x + 2y + 3x$

20) $8k^2 - k - 5k + 7 - 2k^2$

21) Write the perimeter of the figure as a simplified expression

22) Write the perimeter of the figure as a simplified expression

Distributive Property

An expression is in simplest form (simplified) when there no parentheses and no like terms. Use the distributive property to simplify each expression.

23) $2(b - 3)$

24) $x(2y - 5z + 1)$

25) $8(2x - 3) - 6x$

26) $9(2k - 4) - 2(7k - 12)$

27) $-3(-9y - 4) - 2(y - 2)$

Evaluating Expressions (Substitute and Solve)

Evaluate each expression by substituting the values of variables.

28) $3x^2 - 4x$, if $x = 2$

29) $\dfrac{a^2 + b^2}{a - b}$, if $a = 3$ and $b = 4$

30) $n^2 - 3n + 8$, if $n = 4$

Solving Equations (1 and 2 step)

Solve for the variable in each equation.

31) $m + 12 = 10$

32) $-7y = -91$

33) $\dfrac{2}{3}x = 10$

34) $6x + 8 = 50$

35) $13 = -4k + 9$

36) $\frac{x}{2} - 7 = 9$

37) $4 = \frac{w+8}{9}$

38) $2(t - 3) = 12$

Multistep Equations

Solve each equation.

39) $9x + 1 - 7x - 5 = -20$

40) $4m - 5(3m + 10) = 126$

41) $-25 = \frac{1}{2}(10x - 2) + 3x$

42) $-10k + 1 = 40 - 7k$

43) $3(6p - 1) = 11p - 45$

Fractions and Proportions

Simplify each fraction by eliminating common factors in the numerator and denominator.

44) $\dfrac{8}{24}$

45) $\dfrac{91}{27}$

46) $\dfrac{15st}{36t}$

Solve each proportion using the "cross-multiplication" method. Remember to distribute when necessary!

47) $\dfrac{x}{4} = \dfrac{3}{2}$

48) $\dfrac{4}{n+9} = \dfrac{2}{7}$

49) $\dfrac{a}{2} = \dfrac{a-4}{6}$

Justify

The steps required to solve an equation are justified by the properties of equality. They ensure that the equation stays balanced in order to solve for a missing variable.

Addition Property of Equality Add the same value to both sides of an equation	**Subtraction Property of Equality** Subtract the same value from both sides of an equation
Multiplication Property of Equality Multiply the same value to both sides of an equation	**Division Property of Equality** Divide both sides of an equation by the same value

Sometimes you will need to use the **Distributive Property** before you can use the properties of equality.

50) Identify which property justifies each step used to solve the equation below.

Equation Steps	Properties/Reasons
1) $2(x - 9) = -10$	**1)** Given
2) $2x - 18 = -10$	**2)**
3) $2x = 8$	**3)**
4) $x = 4$	**4)**

51) Identify which property justifies each step used to solve the equation below.

Equation Steps	Properties/Reasons
1) $\dfrac{m}{-3} + 10 = -1$	**1)** Given
2) $\dfrac{m}{-3} = -11$	**2)**
3) $m = 33$	**3)**

52) Identify which property justifies each step used to solve the equation below.

Equation Steps	Properties/Reasons
1) $10k - 4 = 2k - 20$	**1)** Given
2) $8k - 4 = -20$	**2)**
3) $8k = -16$	**3)**
4) $k = -2$	**4)**

53) Identify which property justifies each step used to solve the equation below.

Equation Steps	Properties/Reasons
1) $-8(w + 1) = -5(w + 10)$	**1)** Given
2) $-8 - 8 = -5w - 50$	**2)**
3) $-3w - 8 = -50$	**3)**
4) $-3w = -42$	**4)**
5) $w = 14$	**5)**

General Equations

Solve each general equation for the given variable. In other words, isolate the given variable.

54) $A = lw$, solve for w

55) $D = \dfrac{m}{v}$, solve for m

56) $V = \frac{1}{3}Bh$, solve for B

57) $P = 2L + 2W$, solve for W

58) $y = mx + b$, solve for x

59) $E = mc^2$, solve for c

<u>Coordinates and Slope</u>

Write the coordinates for each point on the graph. Remember, a point on the graph has an x-coordinate and y-coordinate, written as an ordered pair in the form (x, y).

60) G _____

61) H _____

62) I _____

63) J _____

64) K _____
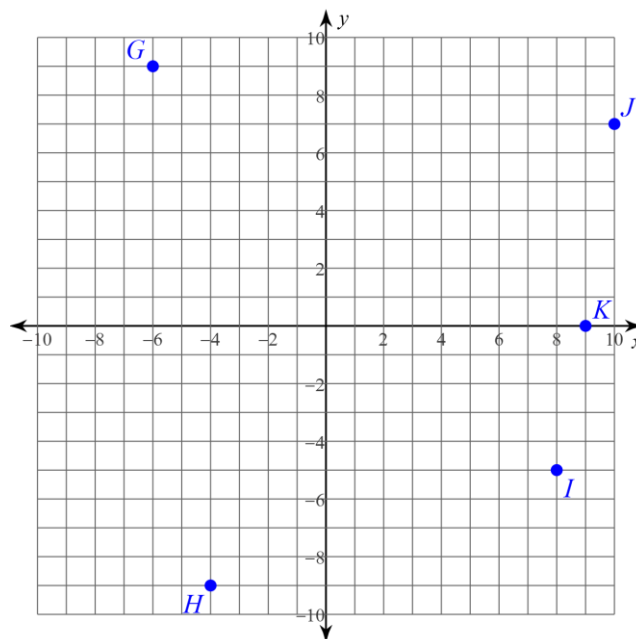
Calculate the slope between each pair of points.

$$\boxed{slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}}$$

65) Slope of line with points H and J (from graph above)

66) Slope of line with points G and K (from graph above)

11

67) Slope of line with points (3,3) and (5,3).

68) Slope of line with points $(-1, 4)$ and $(-1, -2)$

Polygons, Area, and Volume

Match each polygon with its name.

69) _____ Pentagon

70) _____ Decagon

71) _____ Triangle

72) _____ Octagon

73) _____ Heptagon

74) _____ Hexagon

75) _____ Quadrilateral
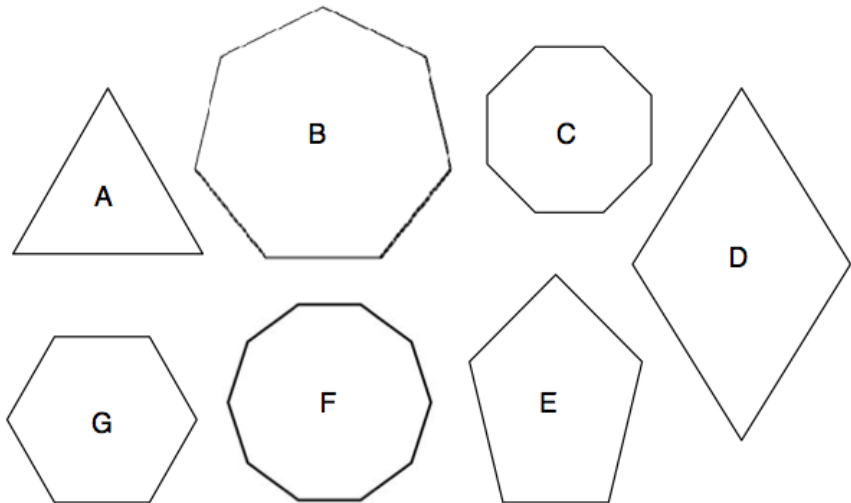
Use the formulas below to solve each problem

Pythagorean Theorem

$$c^2 = a^2 + b^2$$
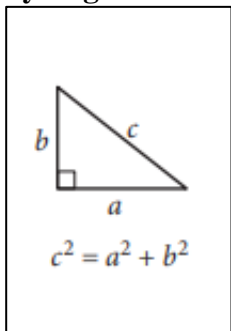
76) Find the missing side length

4 km

3 km

x

Area

$A = \pi r^2$ $A = \ell w$ $A = \frac{1}{2}bh$
$C = 2\pi r$

77) Find the area

6 cm

6 cm
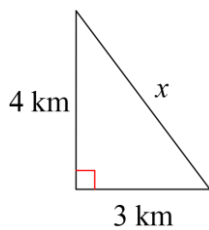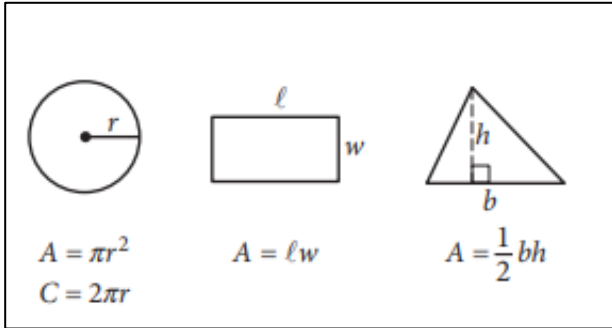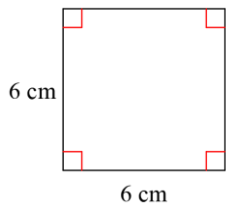
78) Find the area. Round your answer to the nearest tenth.

7.1 km

79) Find the area

12 yd

3.6 yd
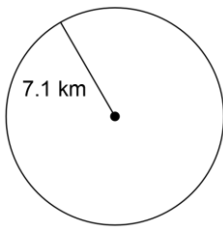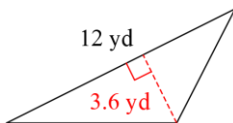
80) Given the area, find the missing length

5.3 in

? in

Area = 32.3 in²

Volume

$V = \ell wh$ $V = \pi r^2 h$ $V = \frac{4}{3}\pi r^3$ $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\ell wh$
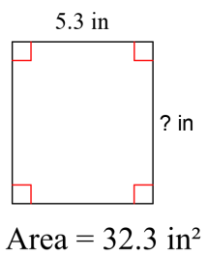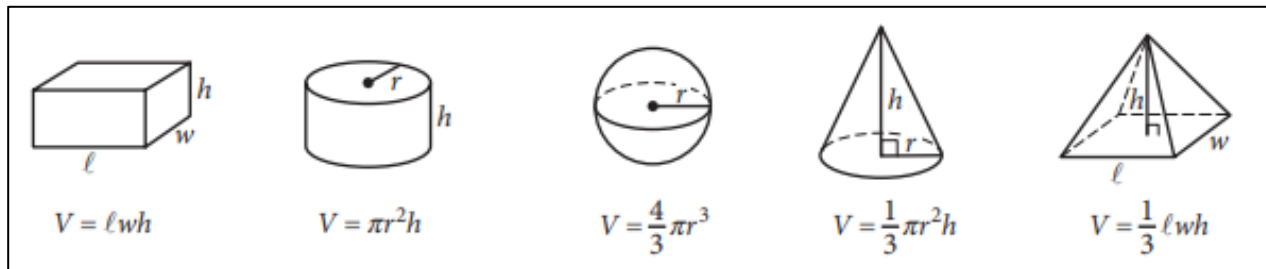
81) Find the volume. Round your answer to the nearest tenth.

8 km

16 km

82) Find the volume.

9 in

7 in   8 in

8 in   7 in

83) Find the volume

7 yd

8 yd   5 yd
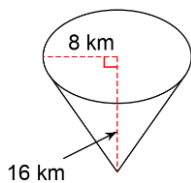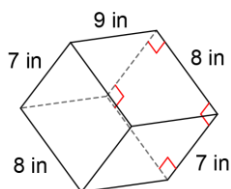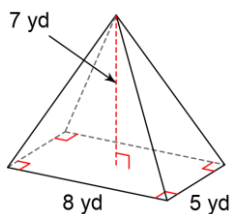
84) A cylindrical dunk tank has a height of eight feet and a radius of six feet. What is the volume of the tank?

85) Pluto has a diameter of 1,477 miles. What is the volume of Pluto? (assume it is a perfect sphere)

Angles and Measurement

Angle Classifications	Examples
Acute angle - Angle with a measure of less than 90°	
Right angle - Angle with a measure of 90°. Two lines that intersect to form a right angle are said to be perpendicular.	
Obtuse angle - Angle with a measure of more than 90° but less than 180°	
Straight angle - Angle with a measure of 180° (straight line)	

Find the measure of each angle, then classify the angle as acute, right, obtuse, or straight.
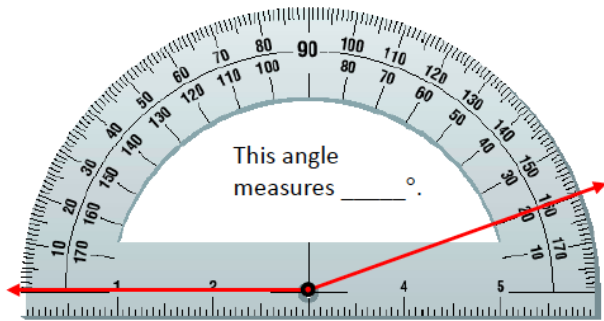
86) Find the measure and classify

This angle measures _____°.
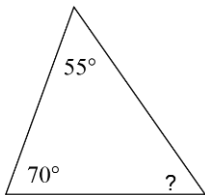
87) Find the measure and classify

This angle measures _____°.
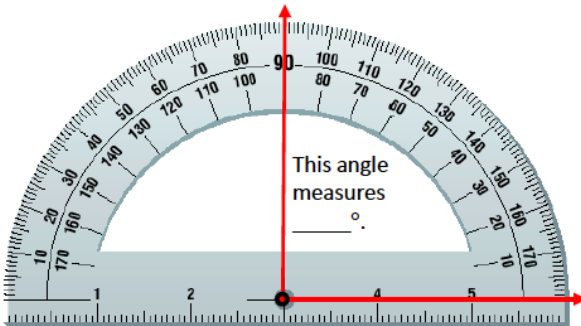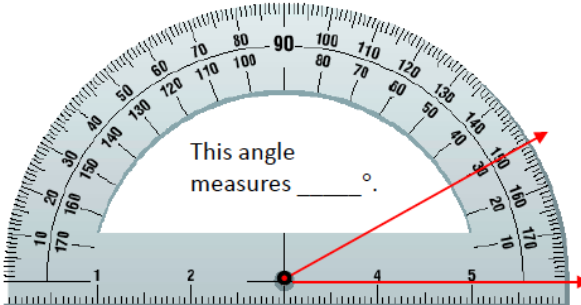
88) Find the measure and classify

This angle measures _____°.

The sum of the angles in a triangle is equal to 180°. Use this information to solve the following problems.

89) Find the missing angle

Word Problems

Set up and solve an equation for each problem.

90) The sum of twice a number and 10 is 36

91) Four times a number divided by 8 is 3

92) The larger of two numbers is four more than the smaller number. If the sum of the numbers is 74, find the two numbers.

93) The highest score on an Algebra test was 42 points more than the lowest score. When added together, the combined score was 154. Find the highest and lowest score.

94) The Washington Redskins scored 24 more than twice the number of points that the Ney York Giants scored. Altogether, the teams scored 66 points. How many points did each team score individually?

95) The sum of two consecutive numbers is 128. Find the two numbers.