Washington Latin A.P. Calculus

2019 Summer Review

!!!! READ CAREFULLY !!!!

Name: _____

INTRODUCTION

I cannot re-teach you Algebra 1, Algebra 2, Geometry and Pre-Calculus. However, if what you learned in those courses is not part of your active working knowledge of mathematics, I cannot teach you Calculus either. Yes, all of the mathematics that you have learned up to this point is going to be drawn upon throughout the year. I often say that **Algebra, Trigonometry and Geometry are the stage on which the ballet of Calculus is performed**. What I mean by that is Calculus is a set of tools that allows us to look at the functions about which we have already learned, in new, exciting and profound ways. Without a solid understanding and facility with those functions and areas of math, the Calculus will be lost on you.

We will begin the year with a brief review of the key concepts and operations that we will see throughout the year. The following are concepts and topics I expect you to know cold:

REVIEW TOPICS

The real number system.

Real numbers, rational numbers, irrational numbers, decimals,
Fractions -- adding, subtracting, multiplying and dividing fractions.
Absolute Value.

Lines

Equations of lines: point-slope and slope-intercept forms.
Graphing lines in the xy-coordinate plane. Slope, the y-intercept and the x-intercept.
Parallel and perpendicular lines.

Systems of linear equations.

Graphing, solving through elimination (combination) and substitution.

Functions

Definition of a function. Domain and range of a function.
Graph of a function. Transformations, reflections and symmetry.
Tables representing functions.
Equations of functions.
Composite functions.
Inverse functions.
Odd and even functions.
Absolute value functions.
Piecewise functions

Quadratics

Definition, standard form, vertex form.
Graphing, finding vertex, axis of symmetry, zeros, y-intercept.
Factoring quadratics. Special cases, difference of squares, perfect square trinomial, quadratics in disguise.
Solving quadratics through factoring, completing the square, and the quadratic formula.

Polynomials

Definition of a polynomial.
Standard form, degree, leading term of a polynomial.
Binomial expansion and Pascale's triangle.
Factoring special cases, *e.g.* difference & sum of cubes.
Graphs: zeros, intercepts, end behavior.

Rational functions

Vertical and horizontal asymptotes.
x and y intercepts.
Zeros.
End behavior.
Holes.

Operations with exponents and exponential functions.

Definitions, negative powers, zero power, fractional powers.
Multiplication rule.
Quotient rule.
Power rule.
Graphs of exponential functions.
e (the coolest of all numbers)

Logarithms and logarithmic functions.

Definition and as inverse of exponential functions.
Natural logarithms and common logarithms.
Multiplication rule.
Quotient rule.
Power rule.
Graphs of logarithmic functions.

Trigonometry

Right triangle trigonometry. SOHCAHTOA. The flagpole problem
The unit circle. Radians.

YOU **MUST KNOW** THE FUNCTION VALUES FOR THE CARDINAL ANGLES. (*to wit,* $\frac{p}{6}, \frac{p}{4}, \frac{p}{3}, \frac{p}{2}, \frac{2p}{3}, \frac{3p}{4}, \frac{5p}{6}, p$, and multiples thereof.)

Inverse trig functions.

Graphs of three primary trig functions, with transformations, *i.e.*

$y = a\ sinb(x - h) + k$ $y = acosb(x - h) + k$ $y = tanx$

Trig identities.

Solving trig equations; general case and restricted case.

Pythagorean triples.

Special triangles; *45º - 45 º - 90 º* and *30 º - 60 º - 90 º*.

Geometry

Triangles and polygons.

Congruence and similarity.

Pythagorean theorem.

Volumes and areas.

Circles.

Cylinders and Cones.

Spheres.

One other thing we need to do is ensure that you are effective at using the graphing calculator. You will need it to solve some exam problems. You will need a **TI - 83**, a **TI - 84** or a comparable graphing calculator. If you have a problem with finding an appropriate calculator, let's figure it out in September, rather than the first week of May, 2019.

REVIEW PROBLEMS

Now, work on the following problems. I will collect these on the first day of class. Bring them with you when you return to school. **I will give a score of 0 - 100, as a test grade, for this review work. The score will not entirely be based on whether your answers are right or wrong. I will also look at whether you follow directions, answer the right question, your thought processes, organization, neatness, demonstrated work and effort.**

You may collaborate with your classmates. You may also use your books, notes and other helpful sources. You should not need to use calculators for any of the problems. You can also email me with questions. (I will be available all summer.) If you just can't figure out the problem, do not stress, but I want to see what you try.

Also, use pencil -- no pens. None, nowhere, nada!!! You will automatically get a *0* if there is ANY pen on your papers, even if it is just your name.

Read the Directions and Problems carefully. To quote a former Latin student, "The hardest thing about Calculus is reading the problem."

PROBLEMS

Real Number System and Basic Algebra

1. Identify four rational and four irrational numbers. For the irrational numbers, show their "exact values" and a decimal approximation.

Rational --

Irrational --

2. What is the mathematical definition of a "rational" number? Why do we call it a "rational" number?

3. In the world of mathematics, what is the difference between an "expression" and an "equation?"

4. Describe what we mean when we say the "solution" to an equation? For example when we say, $x = -21$ is the solution to

$$3x + 4 = 2x - 17$$

what are we saying?

5. What is the Zero Product Rule?

Functions

6. In your own words (or mine), tell me what a function is.

7. In your own words (or mine), what is the inverse of a function.

8. Given the two functions set out in the table below, find:

x	*f(x)*	*g(x)*
-4	*2*	*10*
-3	*5*	*8*
-2	*7*	*5*
-1	*8*	*4*
0	*3*	*3*
1	*12*	*2*
2	*-4*	*0*
3	*-2*	*-1*
4	*0*	*-3*
5	*15*	*-5*

a. f(1) =

b. g(-3) =

c. f(g(5)) =

d. g(f(-3)) =

e. f(f(3)) =

f. $f^{-1}(12)$ =

g. $g^{-1}(-5)$ =

h. g(g(-1)) =

9. Find the inverse of the following function:

$$y = 2x - 9$$

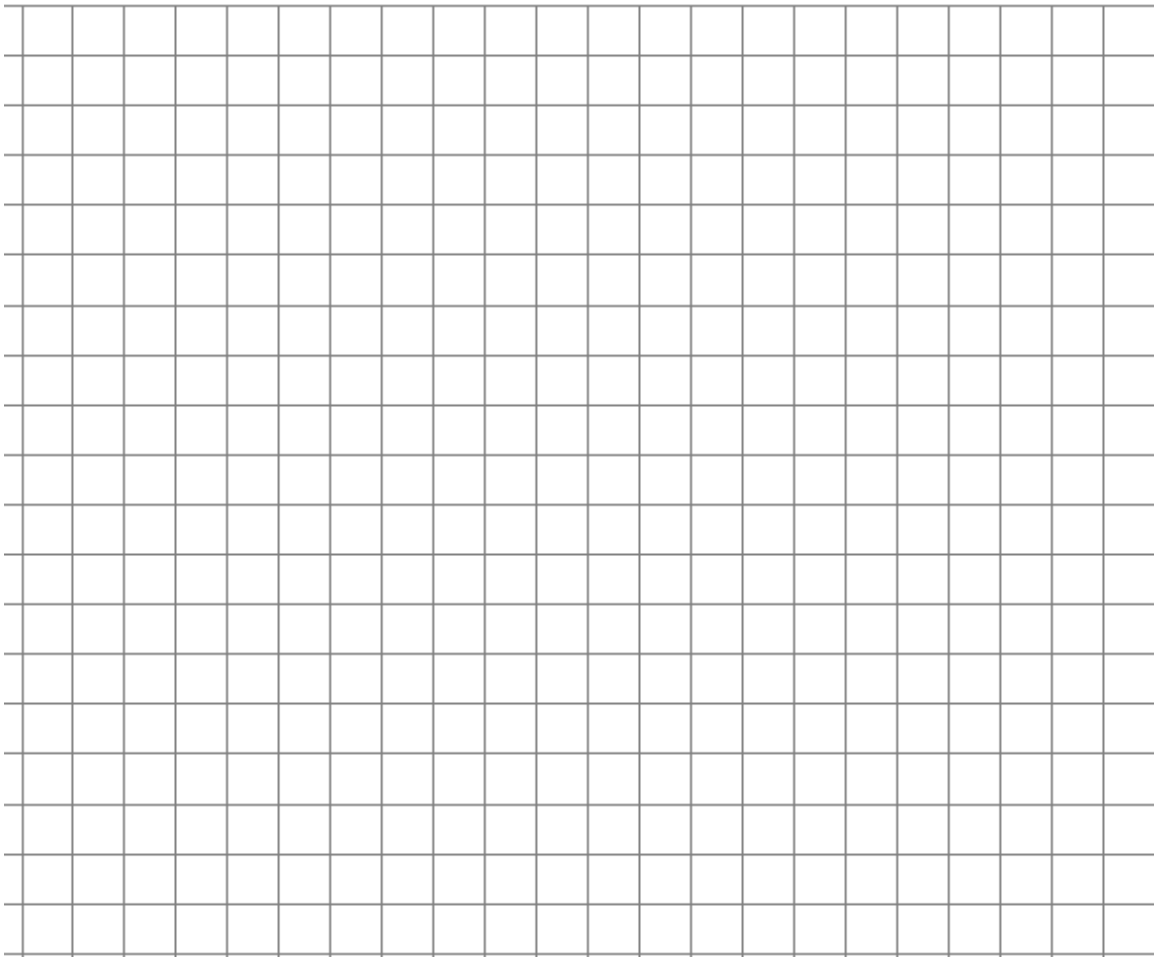
10. a. Is the following function odd or even? Justify your answer algebraically.

$$y = x^2 + 4$$

b. Algebraically, how do we define an odd and an even function?

11. Graph the piecewise function below:

$$f(x) \quad \begin{array}{lll} x+7 & for & x \leq -3 \\ x^2 & for & -3 < x \leq 3 \\ \sqrt{x} & for & 3 < x \end{array}$$



Linear Functions

12. Given the line that passes through the points (-3, 7) and (4, 14);

a. What is the line's equation in slope-intercept form?

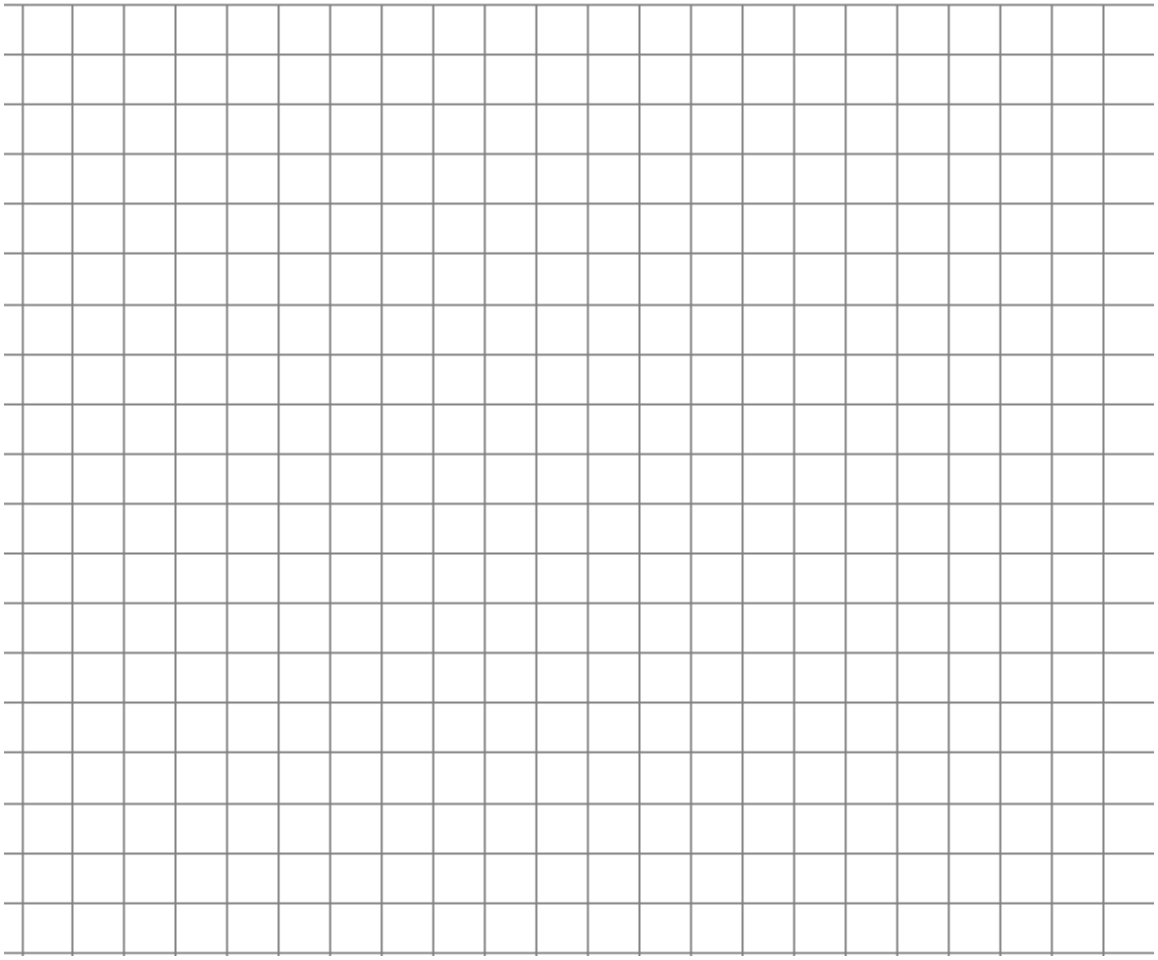
b. What is the line's slope?

c. What is the y-intercept?

d. Write the line's equation in point-slope form.

e. On the following grid, graph the line. Be sure to:
→ label your axes;
→ identify the scale on the grid;
→ use arrows to indicate all infinite curves, including axes.

If you want to use colored pencils on the graph for clarity, OK.



13. What is the equation of the line that is perpendicular to the line graphed above from problem 12, and passes through the point (3, 1). Graph that line as well on the same set of axes.

14. Solve the following for x:

a. $2x+13-7=x+2(31-x)$ b. $|x+13|-3=17$

c. $|2x-14|\pounds 10$ Illustrate your solution on a number line.

d. $x^2-14x+40-13+2x=2x-13+x^2$

Systems of Linear Equations

15. Solve the following systems of equations, using any method:

a. $y=2x-7$ and $y=4x+6$

b. $2x+4y=12$ and $4x-y=6$

c. Where do the curves $y = 3x^2 - 10x + 22$ and $y = 2x^2 - 5x + 36$ intersect?

Exponential Expressions

16. Simplify the following expressions:

a. $10^3 \times 10^2$

b. $\dfrac{a^5 \times a^{-3}}{a^2}$

c. $\dfrac{x^5 \cdot y^{-3} \cdot z^{10}}{x^2 \cdot 3 \cdot z^{-3} \cdot y}$

d. $\dfrac{x^5 y^4}{z^3} \cdot \dfrac{x^2 y^{-4}}{x^7 z^{-3}}$

e. $\dfrac{\cos^3 q}{\sin^2 q} \cdot \tan^3 q \cdot \dfrac{\sec^3 q}{\csc^2 q} \cot^3 q$

f. $\dfrac{\dfrac{a^5}{x^3}}{\dfrac{x^7}{a^2}}$

17. Re-write the following in other than exponential form:

a. a^0 =

b. a^{-1} =

c. $a^{\frac{1}{2}}$ =

d. $a^{\frac{1}{3}}$ =

Quadratics

18. Factor the following expressions:

a. $x^2 + 4x + 3$

b. $x^2 - 5x - 6$

c. $x^2 + 4x - 12$

d. $3x^2 - 12x + 9$

e. $x^2 - 49$

f. $a^2 + 2ab + b^2$

g. $\cos^2 x + 3\cos x + 2$ h. $\cos x - \tan^2 x \cos x$

19. Write the following quadratic expression in standard form.

$2x + 3x^2 - 17x + 32 + 82$

20. What are the *a, b, c* values of the equation in problem *19*, if the general standard form of a quadratic is $ax^2 + bx + c$?

$a =$ $b =$ $c =$

21. Solve for x in whatever way you choose:

a. $x^2 + 7x + 12 = 0$ b. $x^2 - 7x + 12 = 0$

c. $x^2 - 12x - 45 = 0$ d. $4x^2 + 2x - 12 = 0$

e. $x^2 - 4x - 12 = 0$

f. $5x^2 + 6x - 8 = 0$

g. $7(x - 4)(x + 3) = 0$

h. $5(x - 4)^2 - 30 = 0$

22. Use the <u>discriminant</u> ($b^2 - 4ac$) to determine how many solutions there are, and of what kind, to the following quadratic equations:

a. $3x^2 + 6x - 8 = 0$

b. $x^2 + 6x + 9 = 0$

c. $x^2 + 5x + 8 = 0$

23. Follow the steps set out below to graph the following quadratic equation:

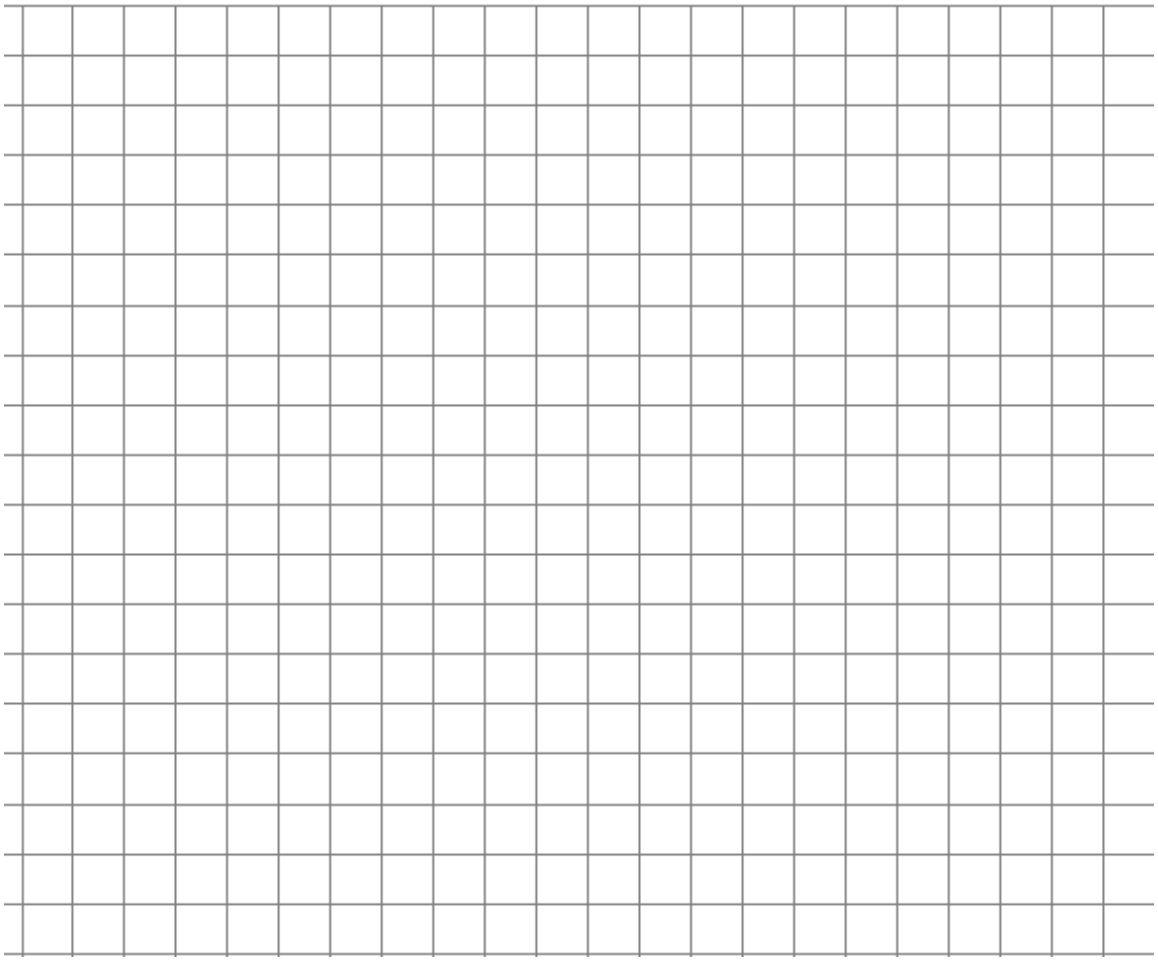
$$y = x^2 - 8x + 1$$

a. First, complete the table of values to the right:

x	y
- 1	
0	
1	
2	
3	
4	

b. find the <u>vertex</u> (x_v, y_v) at $x_v = -\frac{b}{2a}$ and $y_v = f(x_v)$.

c. Graph the curve below. Remember, <u>label axes</u> and <u>include a scale</u> and <u>arrows</u>.



What is the curve's axis of symmetry?

24. Where, in the *xy* - coordinate plane, is the graph of the equation

$$y = (x - 5)^2 + 3$$

relative to its parent function, $y = x^2$?

What are the coordinates of the curve's vertex?

Logarithms

25. What is a logarithm?

26. Simplify the following:

a. $\log_2 16 =$

b. $\log_8 1 =$

c. $\log_6 216 =$

d. $\log 1{,}000 =$

e. $2\log x + 4\log y - \frac{1}{2}\log z =$

f. $\log_4 256 =$

g. $\ln e =$

h. $\ln e^4 =$

i. $e^{\ln 42} =$

j. $\ln \frac{1}{e^2} =$

k. $\log_3 \sqrt{81} =$

l. $\log_6 \sqrt{6^3} =$

27. Re-write the following as a logarithmic equation if exponential, and as an exponential equation if logarithmic:

a. $5^x = 37$

b. $\log 3{,}527 = n$

c. $\ln e^2 = 2$

d. $\log 35 - \log x = 2.73$

e. $e^2 e^x = 5.27$

f. $\log_4 N = \frac{1}{5}$

28. Solve the following for the unknown:

a. $\log_2(x^2 + 2x) = 3$

b. $\frac{3}{2}\log_2 4 - \frac{1}{2}\log_2 x = 3$

c. $\log_8(2x - 1) = \frac{1}{3}$

d. $\log 4 + 2\log x = 2$

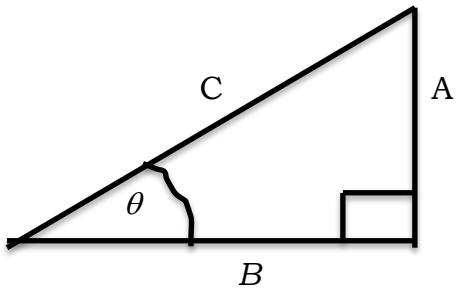
Trigonometry

29. Given the right triangle below, what is the definition of

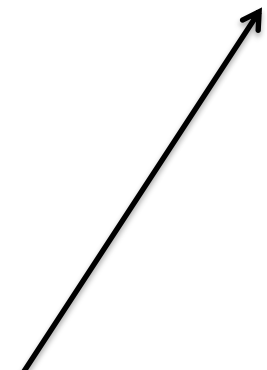
sin θ = *cot* θ =

tan θ = *csc* θ =

cos θ = *sec* θ =



30. In the unit circle at the right,
what are: ((*a, b)* are the coordinates of the intersection



of the ray and the circle.)

$\sin\theta =$

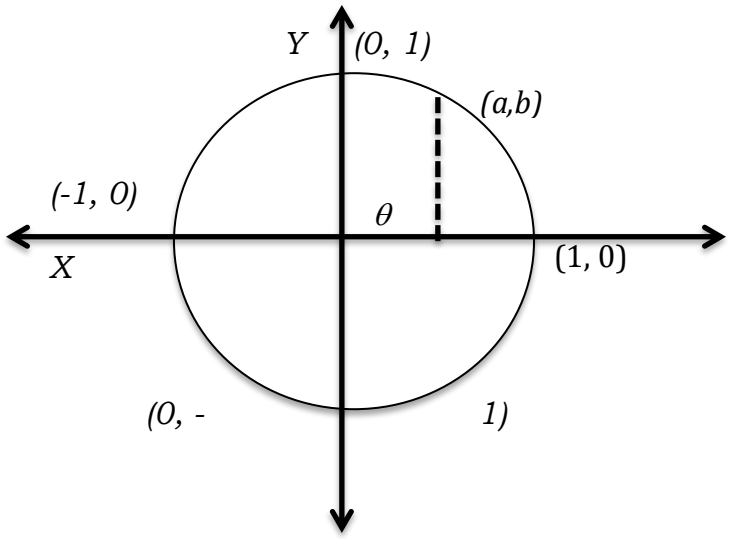
$\cos\theta =$

$\cot\theta =$

$\tan\theta =$

$\csc\theta =$

$\sec\theta =$



31. Complete the table with <u>exact</u> values. **Exact** value means **NOT in decimal form**.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0			
$\frac{p}{6}$			
$\frac{p}{4}$			
$\frac{p}{3}$			
$\frac{p}{2}$			
$\frac{2p}{3}$			
$\frac{3p}{4}$			
$\frac{5p}{6}$			

p			

32. There are 3 important Pythagorean identities in trigonometry. The first is:

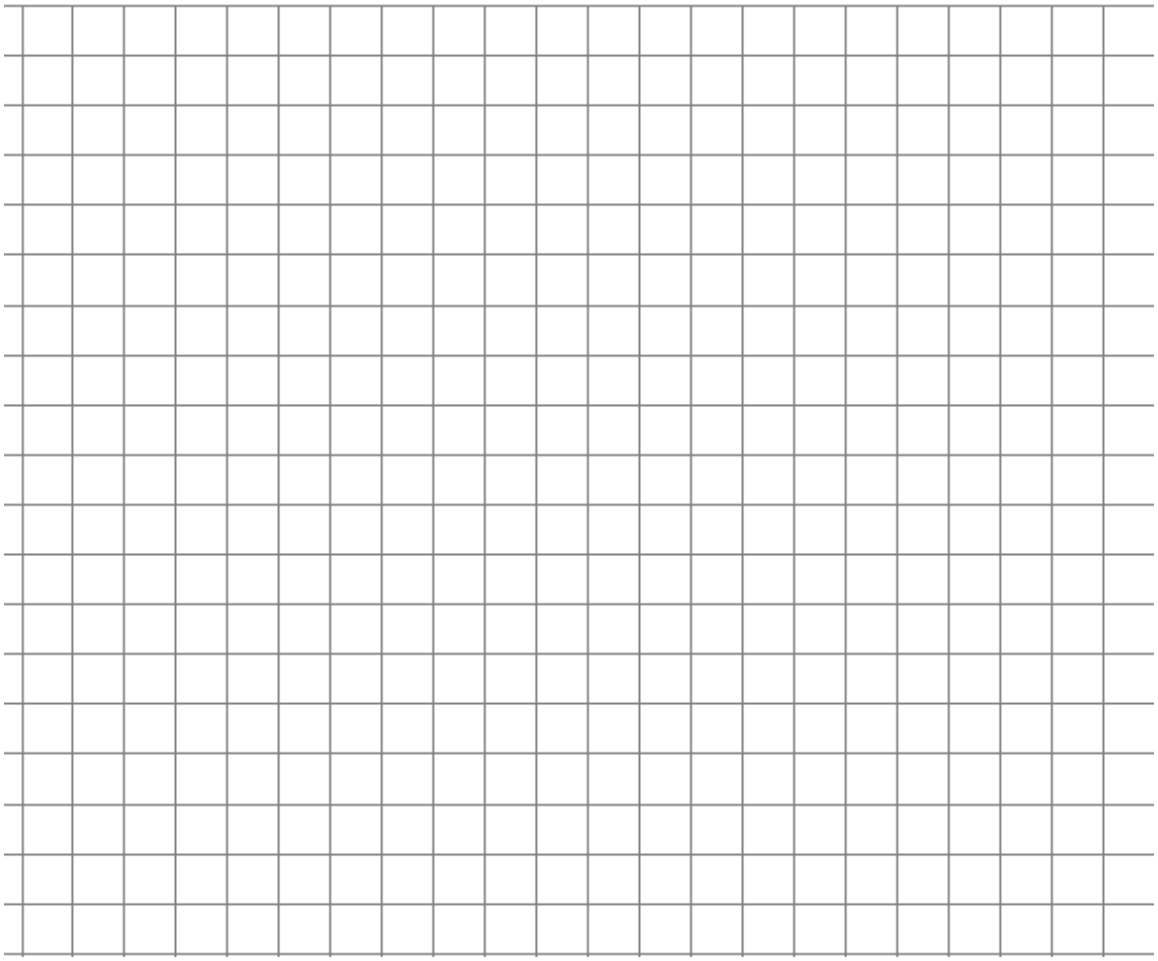
$$\sin^2 q + \cos^2 q = 1$$

What are the other two Pythagorean identities (that don't include either the sine or cosine)?

33. Prove in any way you choose that for any angle theta;

$$\sin^2 q + \cos^2 q = 1$$

34. On the grid below, draw a sine curve and a cosine curve. Use a radian scale on the x - axis. Make the graphs so that I can clearly interpret the period and amplitude of the two curves. Label axes, show the scale and use arrows. Colored pencils are OK, to distinguish the two curves.



What are the period and the amplitude of your sine and cosine curves?

	sine	cosine
period	--	
amplitude	--	

35. Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = \frac{5}{13}$. (if uncertain about how to do this, begin by drawing a right triangle and showing as much information as you can on the diagram!!)

36. Find $cos\theta$ and $tan\theta$ if $sin\theta = \frac{8}{17}$. Again, draw a right triangle.

37. You are at the beach, flying a kite. Your father wants you to figure out how high the kite is flying. You have let out one whole ball of string, which is 130 feet of string. Your father tells you that the kite string is at an angle of 67˚ to the ground. Using your calculator for trig function values, calculate how high your kite is. (Include a diagram and show each step in your work.)

38. Prove the following identities:

a. $(1 - sinx)(1 + sinx) = cosx$ b. $cosx + sinxtanx = secx$

c. $sin^2x - sin^4x = cos^2x - cos^4x$ d. $\frac{1}{tanx} + tanx = \frac{sec^2x}{tanx}$

39. Find all solutions in the interval $0 \leq x < 2\pi$:

a. $2\cos x \sin x = \sin x$

b. $-\sin x + 1 = 2\cos^2 x$

40. Find the general solution to the following:

a. $\sin 3x = \frac{\sqrt{3}}{2}$

b. $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

Rational Expressions and Functions

41. Simplify the following rational expressions:

a. $\dfrac{4x^2}{4x^3 + 12x}$

b. $\dfrac{x^2 - 9}{x + 3}$

c. $\dfrac{x^2 - 2x - 3}{x^2 - 7x + 12}$

d. $\dfrac{x^3 - 5x^2 + 6x}{x^2 - 4} \times \dfrac{x^2 + 3x + 2}{x^3 - 2x^2 - 3x}$

42. Perform the following operations:

a. $\dfrac{3}{4x} - \dfrac{2}{x^2}$

b. $\dfrac{\dfrac{2}{x} + \dfrac{3}{y}}{\dfrac{-5}{x} + \dfrac{7}{y}}$

c. $\frac{3}{x+1}+\frac{x}{x-1}$

d. $\frac{4}{x^2-9}+\frac{7}{x+3}$

e. $\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}$

f. $\frac{5}{x^2-3x+2}+\frac{2}{x-1}+\frac{4}{x-2}$

43. On the following page, graph the rational function. Identify the elements you use to construct the graph.

$$y = \frac{x^2 - 6x + 8}{x^2 - x - 12}$$

a. What are the vertical asymptotes?

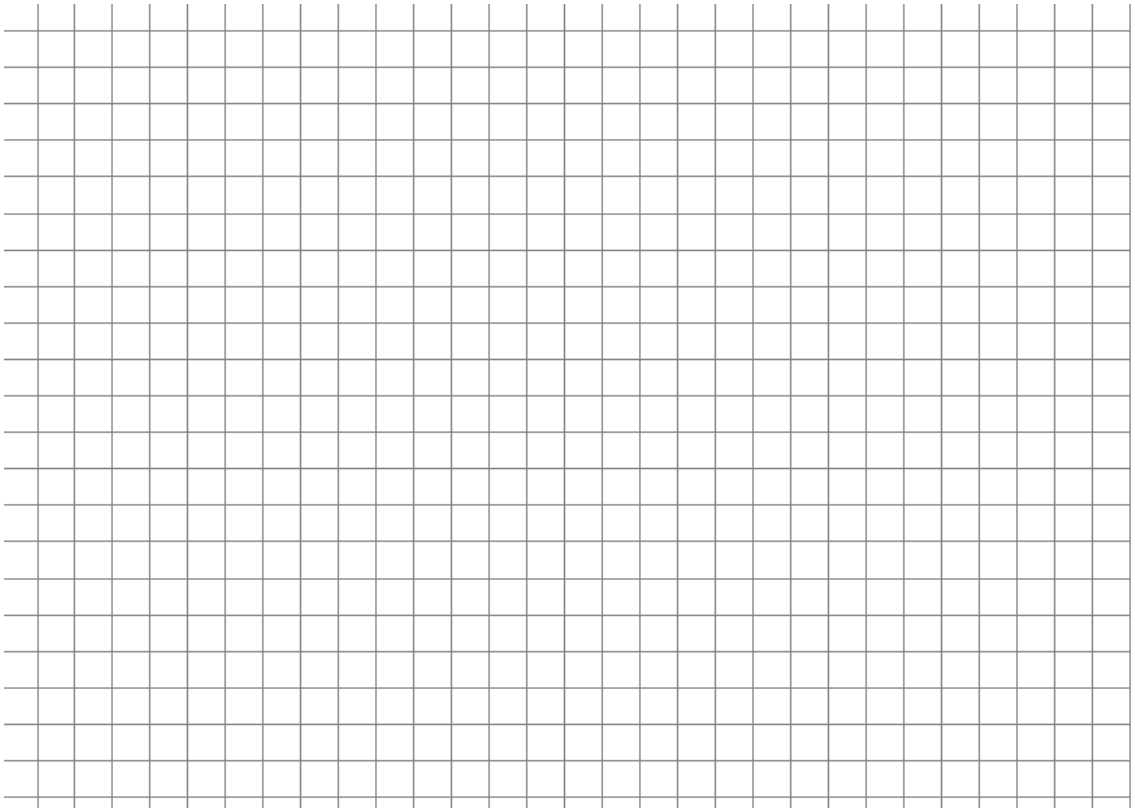
b. What horizontal asymptotes are there?

c. What is the y - intercept?

d. What are the curve's zeroes?

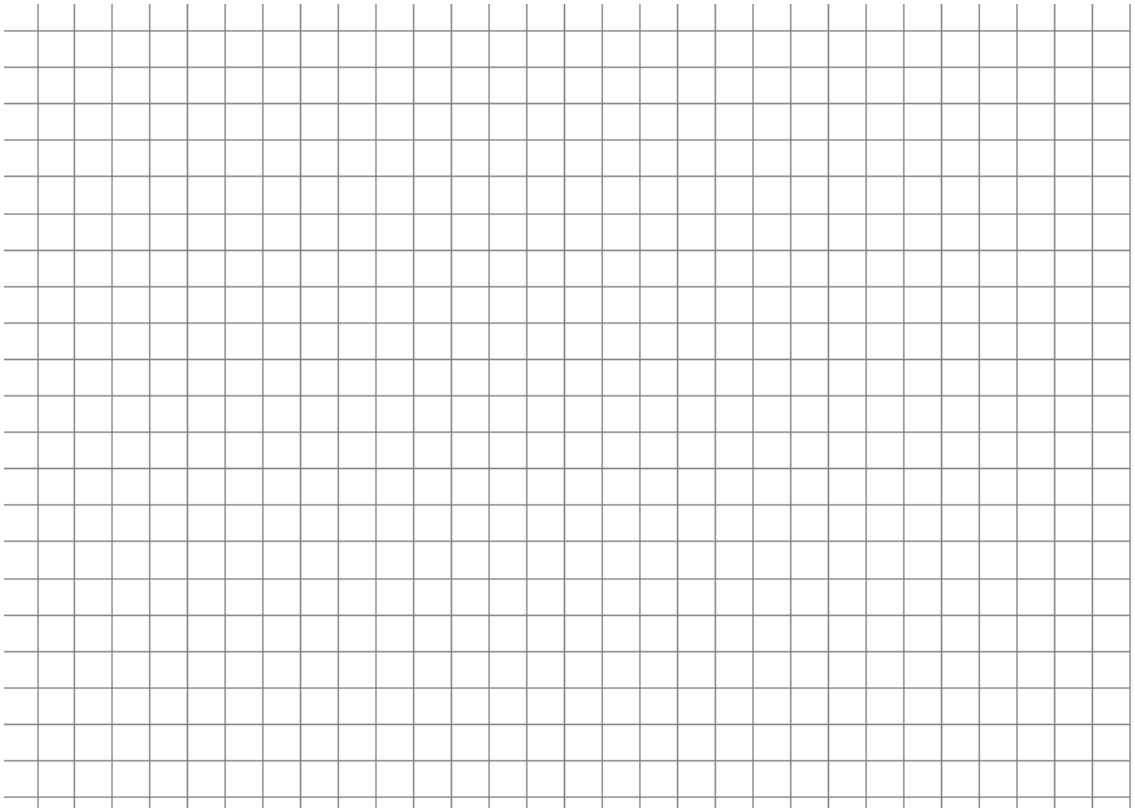
e. What happens to the curve, $y = f(x)$, as $x \to -\infty$ and as $x \to \infty$?

f. Are there any holes? If so, where. Justify your answer.



44. Graph:

$$\boldsymbol{y} = \frac{x^2 - 8x + 12}{x(x^2 - x - 20)}$$



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f. Are there any holes? If so, where. Justify your answer.

Topics in Geometry

45. What are the area and the circumference of a circle in terms of its radius?

46. What are the volumes of a cone and a sphere? What is the surface area of each?

47. We all know that $\pi = 3.1415926\ldots$. What is the definition of π? How did mathematicians figure out its value?

48. What are the areas of a square, a rectangle, a parallelogram, a trapezoid and a triangle?

49. What do we mean when we say that geometric shapes are similar?

50. If the side of a 45^o- 45^o- 90^o right triangle is 10, what is the length of the hypotenuse?

51. In a 30^o- 60^o- 90^o right triangle, if the short side is a, what are the other two sides?

52. In an isosceles triangle, one angle has a measure of 36°. What are the other two angles if the given angle is opposite one of the two equal sides?

53. What are the squares of the numbers 1 - 15?

1 - 2 - 3 - 4 - 5 -

6 - 7 - 8 - 9 - 10 -

11 - 12 - 13 - 14 - 15 -

54. What are the divisibility rules for,

2 -

3 -

4 -

5 -

6 -

9 -

55. What is the hypotenuse of the right triangle with the following pairs of legs?

3, 4 5, 12 7, 24 8, 15

GOOD WORK!!! I Can't wait to get started with Calculus (real mathematics). See you soon.